Rule of Mixture for Composites

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Introduction

- Mechanical properties of composites depends on the volume fraction of reinforcement and matrix.
- The basic properties can be calculated using rule-of-mixture principle with some assumptions
- The type of reinforcement (fiber, particle or whiskers) and their orientation play a major role in determining the strength of composites
- In composites, if fibers are oriented at an angle, their strength along the fiber direction will be more than the other directions
- In particle reinforced composites, distribution of particle through out the component is key to provide uniform material property

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Volume Fraction

Volume fraction of fiber, v_f = volume of fiber/ volume of composite

Volume of composite, $v_c = v_f + v_m$

For unit volume of composite, $1 = v_f + v_m$

Volume fraction of matrix, $v_m = 1 – v_f$

<u>Notations used in rule-of-mixture are as follows:</u>

c, f, m represent composite, fiber, matrix respectively, V is Volume fraction, P is load withstand, A is cross sectional area, E is Elastic modulus, σ is stress, ε is Strain, μ is Poisson ratio and ρ is Density

Rule of Mixture

When a fiber oriented composite is loaded in the longitudinal direction of fiber alignment then the strain in the fiber, matrix, and composite are equal (iso-strain).

For iso-strain condition, $\varepsilon_c = \varepsilon_f = \varepsilon_m$

Load withstand by composite, $P_c = P_f + P_m$

Load withstand can also be written as, $\sigma_c A_c = \sigma_f A_f + \sigma_m A_m$

Stress of composite for longitudinal load, $\sigma_c = \sigma_f v_f + \sigma_m v_m$

Longitudinal Elastic Modulus of composite, $E_{cL} = E_f v_f + E_m v_m$

Poisson ratio of composite, $\mu_{12} = v_f \mu_f + v_m \mu_m$

Density of composite, $\rho_c = \rho_f v_f + \rho_m v_m$

Fraction of load taken by fiber, $\frac{P_f}{P_c} = \frac{\sigma_f v_f}{\sigma_c} = \frac{E_f v_f}{E_c}$

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Rule of Mixture cont....

When a fiber oriented composite is loaded in the transverse direction of fiber alignment then the stress in the fiber, matrix, and composite are equal (iso-stress).

For iso-stress condition, $\sigma_c = \sigma_f = \sigma_m$

Strain of the composite, $\varepsilon_c = \varepsilon_f v_f + \varepsilon_m v_m$

Transverse elastic modulus of composite, $\frac{1}{E_{cT}} = \frac{v_f}{E_f} + \frac{v_m}{E_m}$

Rule of Mixture cont....

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- In particle reinforced composite the elastic modulus shall fall between a upper and lower values as per volume fraction.
- The particles are assumed to be evenly distributed throughout the matrix

Upper Elastic Modulus of composite, $E_{c\ Upper} = E_p v_p + E_m v_m$

Lower Elastic Modulus of composite, $E_{c\ Lower} = \dfrac{E_p E_m}{E_m v_p + E_p v_m}$

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Tutorial 1

Calculate the longitudinal modulus and tensile strength of a unidirectional composite containing 55 percent by volume of Sisal fibers in epoxy matrix. The modulus and strength of fiber is 30 GPa and 600 MPa respectively and the same for matrix is 3.5 GPa and 100 MPa respectively. Find the fraction of load taken by fibers in the composite.

Solution:

Volume fraction of fiber, $V_f = 0.55$ or 55 %

Volume fraction of matrix, $V_m = 1 – 0.55 = 0.45$ or 45 %

Longitudinal Modulus of composite, $E_{cL} = 30 \times 0.55 + 3.5 \times 0.45 = 18.1$ GPa

Longitudinal strength of composite , $\sigma_c = 600 \times 0.55 + 100 \times 0.45 = 375$ MPa

Fraction of load taken by fiber, $$\frac{P_f}{P_c} = \frac{600 \times 0.55}{375} = 0.88 \text{ or } 88\%$$

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Tutorial 2

A metal-Matrix composite is made from a Boron fibre reinforced in Aluminium alloy. To form the Boron fibre, a Tungsten wire (r=10 μm) is coated with Boron, giving a final radius of 75μm. The Aluminium alloy is then bonded around the Boron fibres, given a volume fraction of 0.65 for the Aluminium alloy. Assuming that rule of mixtures applies also to ternary mixtures; calculate the effective tensile elastic modulus of composite material under isostrain conditions. Given $E_{Tungsten}$ = 410GPa; E_{Boran} = 379GPa and $E_{Aluminium}$= 68.9GPa.

<u>Solution:</u>

Assume unit length of fiber and find the volume fraction of tungsten wire

Volume fraction of Tungsten wire, $V_T = 10/75 = 0.133$

Elastic Modulus of Boron fiber, $E_f = 410 \times 0.133 + 379 \times (1 - 0.133) = 383$ GPa

Elastic Modulus of composite, $E_c = 383 \times (1 - 0.65) + 68.9 \times 0.65 = 178.8$ GPa

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Thank you

Reference:
1. Fiber Reinforced Composites by P K Mallick
2. Materials Science and Engineering by William D Callister