Mechanics of Fiber Reinforced Composites

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Reference: Fiber Reinforced Composites by P K Mallick

Introduction

- Mechanics of materials deal with stresses, strains, and deformations in structures subjected to mechanical loads
- The fiber-reinforced composite material is assumed to be homogeneous and equations of orthotropic elasticity are used
- The following assumptions are considered
    1. Fibers are uniformly distributed throughout the matrix.
    2. Perfect bonding exists between the fibers and matrix.
    3. Matrix is free of voids.
    4. The applied force is either parallel to or normal to the fiber direction.
    5. Both fibers and matrix behave as linearly elastic materials.

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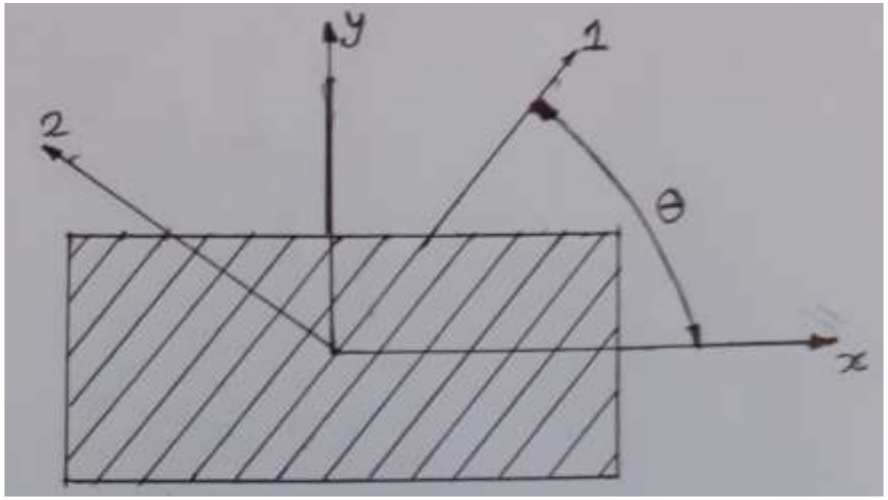
Fiber Reinforced Lamina

- Consider a thin lamina with fibers arranged parallel to each other in a matrix as shown in figure.
- Angle between the loading direction and the fiber direction is called fiber orientation angle θ
- In a 0^o lamina, the principal fiber axis 1 coincides with the loading axis x, but in a 90^o lamina, the principal fiber axis 1 is at 90^o angle with the loading axis x.



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Elastic Properties of a Lamina

- Young's modulus E, Poisson's ratio ν and shear modulus G
- In a isotropic material, properties are same in all the direction. G can be represented in terms of E & ν . Hence two independent properties
- In a anisotropic material, properties are different in all the directions. It has 21 independent elastic constants
- Orthotropic material has nine independent elastic constants

  E_{11}, E_{22}, E_{33}, G_{12}, G_{13}, G_{23}, ν_{12}, ν_{13}, ν_{23}

Notation: Lamina properties are denoted by two subscripts. First subscript represents the loading direction and the second subscript represents the direction in which the particular property is measured.

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Continuous fiber 0° lamina

In a 0° lamina, fiber direction 1 and material direction x are same

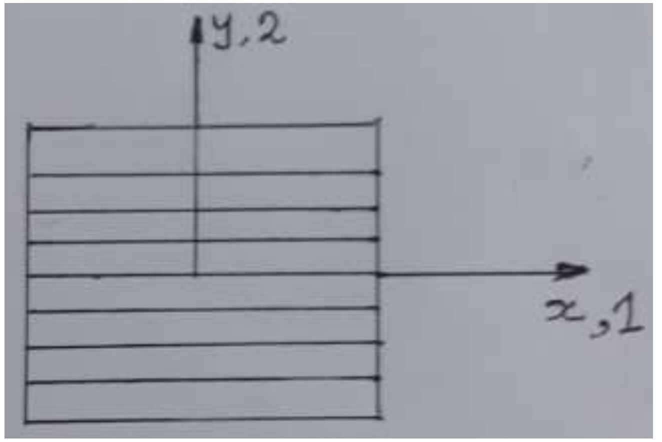
Longitudinal modulus, $$E_{11} = E_f v_f + E_m v_m$$

Transverse modulus , $$E_{22} = \frac{E_f E_m}{E_f v_m + E_m v_f}$$

Major Poisson's ratio, $$\nu_{12} = \nu_f v_f + \nu_m v_m$$

Minor Poisson's ratio, $$\nu_{21} = \frac{E_{22}}{E_{11}} \nu_{12}$$

In-plane shear modulus, $$G_{12} = G_{21} = \frac{G_f G_m}{G_f v_m + G_m v_f}$$



Four independent elastic constants E_{11}, E_{22}, ν_{12}, and G_{12} are required to describe the in-plane elastic behavior of a lamina

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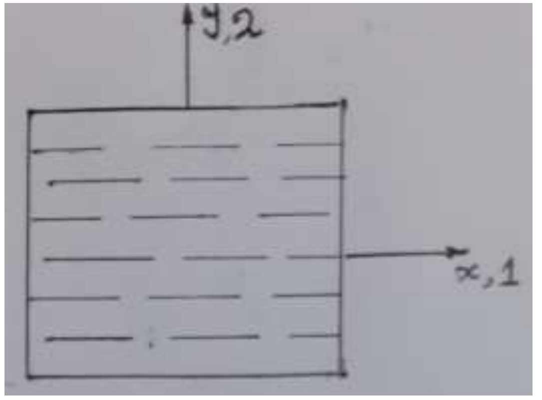
Discontinuous fiber 0º lamina

Longitudinal modulus, $$E_{11} = \frac{1 + 2(l_f/d_f)\eta_L v_f}{1 - \eta_L v_f} E_m$$

Transverse modulus , $$E_{22} = \frac{1 + 2\eta_T v_f}{1 - \eta_T v_f} E_m$$

In-plane shear modulus, $$G_{12} = G_{21} = \frac{1 + \eta_G v_f}{1 - \eta_G v_f} G_m$$



Where, $$\eta_L = \frac{(E_f/E_m) - 1}{(E_f/E_m) + 2(l_f/d_f)} \quad \eta_T = \frac{(E_f/E_m) - 1}{(E_f/E_m) + 2} \quad \eta_G = \frac{(G_f/G_m) - 1}{(G_f/G_m) + 1}$$

Major & Minor Poisson's ratio are same as continuous fiber lamina

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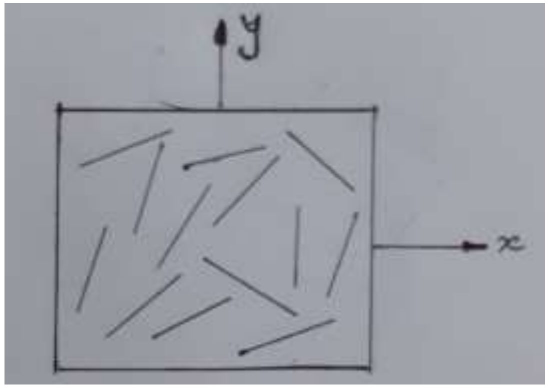
Random Discontinuous lamina

- A thin lamina containing randomly oriented discontinuous fiber exhibits planar isotropic behaviour
- Elastic properties are same in all the direction
- The properties shall be calculated from longitudinal and transverse modulus of unidirectional discontinuous fiber 0° lamina

Young's modulus, $$E_{random} = \frac{3}{8}E_{11} + \frac{5}{8}E_{22}$$

Shear modulus , $$G_{random} = \frac{1}{8}E_{11} + \frac{1}{4}E_{22}$$

Poisson's ratio, $$\nu_{random} = \frac{E_{random}}{2G_{random}} - 1$$



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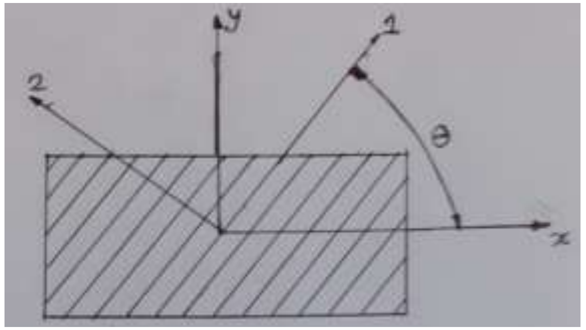
Continuous Angle-ply lamina

The elastic properties of angle ply lamina in which the continuous fibers are aligned at an angle θ with the x axis are as follows



$$\frac{1}{E_{xx}} = \frac{cos^4\theta}{E_{11}} + \frac{sin^4\theta}{E_{22}} + \frac{1}{4}\left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}}\right)sin^2 2\theta$$

$$\frac{1}{E_{yy}} = \frac{sin^4\theta}{E_{11}} + \frac{cos^4\theta}{E_{22}} + \frac{1}{4}\left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}}\right)sin^2 2\theta$$

$$\frac{1}{G_{xy}} = \frac{1}{E_{11}} + \frac{2\nu_{12}}{E_{11}} + \frac{1}{E_{22}} - \left(\frac{1}{E_{11}} + \frac{2\nu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right)cos^2 2\theta$$

$$\nu_{xy} = E_{xx}\left[\frac{\nu_{12}}{E_{11}} - \frac{1}{4}\left(\frac{1}{E_{11}} + \frac{2\nu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right)sin^2 2\theta\right]$$

$$\nu_{yx} = \frac{E_{yy}}{E_{xx}}\nu_{xy}$$

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Stress & Strain Transformation

In stress analysis of thin lamina, it is necessary to transform stresses in the loading direction to fiber direction.

Stress transformation equations are as follows

$$\sigma_{11} = \sigma_{xx} cos^2\theta + \sigma_{yy} sin^2\theta + 2\tau_{xy} cos\theta sin\theta$$

$$\sigma_{22} = \sigma_{xx} sin^2\theta + \sigma_{yy} cos^2\theta - 2\tau_{xy} cos\theta sin\theta$$

$$\tau_{12} = 2(-\sigma_{xx} + \sigma_{yy}) sin\theta cos\theta + \tau_{xy}(cos^2\theta - sin^2\theta)$$

Strain transformation equations are as follows

$$\varepsilon_{11} = \varepsilon_{xx} cos^2\theta + \varepsilon_{yy} sin^2\theta + 2\gamma_{xy} cos\theta sin\theta$$

$$\varepsilon_{22} = \varepsilon_{xx} sin^2\theta + \varepsilon_{yy} cos^2\theta - 2\gamma_{xy} cos\theta sin\theta$$

$$\gamma_{12} = 2(-\varepsilon_{xx} + \varepsilon_{yy}) sin\theta cos\theta + \gamma_{xy}(cos^2\theta - sin^2\theta)$$

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Tutorial 1

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A square composite plate containing unidirectional continuous E glass fiber-reinforced epoxy is subjected to a uniaxial tensile load of 1000 N. The length and width of the plate are 100 mm each and thickness is 1 mm. Take: E_{11} = 138 GPa, E_{22} = 12 GPa, ν_{12} = 0.21. Calculate the changes in length and width of the plate when load is applied

a) Parallel to the fiber direction

b) Normal to the fiber direction

Solution: a) Parallel to fiber direction

Stress in the fiber direction, $$\sigma_{11} = \frac{1000}{100 \text{ x } 1} = 10 \text{ MPa}$$

Strain in the fiber direction, $$\varepsilon_{11} = \frac{\sigma_{11}}{E_{11}} = \frac{10 \text{ MPa}}{138 \text{ GPa}} = 0.0000725$$

Strain in the matrix direction, $$-\varepsilon_{22} = \nu_{12}\varepsilon_{11} = 0.21 \text{ x } 0.0000725 = 0.0000152$$

Change in length, $$\delta L = L\varepsilon_{11} = 100 \text{ x } 0.0000725 = 0.00725 \text{ mm}$$

Change in width, $$\delta W = W\varepsilon_{22} = 100 \text{ x} - 0.0000152 = -0.00152 \text{ mm}$$

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Tutorial 1 cont….

Solution: b) Normal to fiber direction

Poisson's ratio, $\nu_{21} = \frac{E_{22}}{E_{11}} \nu_{12} = \frac{12}{138} \text{ x } 0.21 = 0.0183$

Stress in the matrix direction, $\sigma_{22} = \frac{1000}{100 \text{ x } 1} = 10 \text{ MPa}$

Strain in the matrix direction, $\varepsilon_{22} = \frac{\sigma_{22}}{E_{22}} = \frac{10 \text{ MPa}}{12 \text{ GPa}} = 0.0008$

Strain in the fiber direction, $-\varepsilon_{11} = \nu_{21}\varepsilon_{22} = 0.0183 \text{ x } 0.0008 = 0.0000146$

Change in length, $\delta L = L\varepsilon_{11} = 100 \text{ x} - 0.0000146 = -0.00146 \text{ mm}$

Change in width, $\delta W = W\varepsilon_{22} = 100 \text{ x}0.0008 = 0.08 \text{ mm}$

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Tutorial 2

A unidirectional discontinuous fiber lamina contains T-300 carbon fiber in an epoxy matrix. The fiber aspect ratio (l_f/d_f) is 50, and the fiber volume fraction is 0.5. Determine the elastic constants E_{11}, E_{22}, ν_{12}, ν_{21}, and G_{12} for the lamina.

The properties for fiber, E_f=345 Gpa, ν_f=0.21 and for matrix, E_m=2.07 GPa and ν_m=0.45.

Solution:

Constants, E_f/E_m = 166.6 $\eta_L = \frac{166.6 - 1}{166.6 + 2(50)} = 0.621$ $\eta_T = \frac{166.6 - 1}{166.6 + 2} = 0.982$

Elastic Modulus in the fiber direction, $E_{11} = \frac{1 + 2(50)0.621x0.5}{1 - 0.621x0.5} 2.07 = 97.57\ \text{GPa}$

Elastic Modulus in the matrix direction, $E_{22} = \frac{1 + 2x0.982x.5}{1 - 0.982x.5} 2.07 = 8.06\ \text{GPa}$

Major Poisson's ratio, $\nu_{12} = 0.21x0.5 + 0.45x(1 - 0.5) = 0.33$

Minor Poisson's ratio, $\nu_{21} = \frac{8.06}{97.57} 0.33 = 0.027$

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Tutorial 2 cont….

Solution:

Values of G_f and G_m is not available, let us assume isotropic relations to find them. This will introduce some error.

Shear Modulus of fiber, $$G_f = \frac{E_f}{2(1+\nu_f)} = \frac{345}{2(1+0.21)} = 142.6 \text{ GPa}$$

Shear Modulus of matrix, $$G_m = \frac{E_m}{2(1+\nu_m)} = \frac{2.07}{2(1+0.45)} = 0.714 \text{ GPa}$$

Constant, $$\eta_G = \frac{(142.6/0.714)-1}{(142.6/0.714)+1} = 0.99$$

Shear Modulus, $$G_{12} = G_{21} = \frac{1+0.99\text{x}0.5}{1-0.99\text{x}0.5}0.714 = 2.11 \text{ GPa}$$

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Tutorial 3

A carbon fiber-epoxy lamina with fiber orientation angle of 45° is subjected to a biaxial stress state of σ_{xx} = 100 MPa and σ_{yy} = 50 MPa. Determine the stresses in the 1–2 directions. Assume $\tau_{xy} = 0$

<u>Solution:</u>

Stress in the fiber direction, $\sigma_{11} = \sigma_{xx}cos^2\theta + \sigma_{yy}sin^2\theta$

$$\sigma_{11} = 100cos^2 45 + 50sin^2 45 = 75\text{MPa}$$

Stress in the matrix direction, $\sigma_{22} = \sigma_{xx}sin^2\theta + \sigma_{yy}cos^2\theta$

$$\sigma_{22} = 100sin^2 45 + 50cos^2 45 = 75\text{MPa}$$

Shear Stress in the plane, $\tau_{12} = 2(-\sigma_{xx} + \sigma_{yy})sin\theta cos\theta$

$$\tau_{12} = 2(-100 + 50)sin45cos45 = 50\text{MPa}$$

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Thank you

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