

Engineering Thermodynamics

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Units and Parameters

Fundamental units:

Mass	kilograms	kg
Length	meter	m
Time	seconds	s
Temperature	Celcius/Kelvin	°C / K

Derived units:

Newton's Law states: Force = mass x acceleration

$$F = m \cdot a$$

$$[N] = [kg] \cdot [m/s^2] \quad \rightarrow 1 N = 1 kg \cdot m/s^2$$

$$P = F / A$$

$$[Pa] = [kg \cdot m/s^2] / [m^2] \quad \rightarrow 1 Pa = 1 N/m^2$$

$$W = F \cdot x$$

$$[J] = [kg \cdot m/s^2] \cdot [m] \quad \rightarrow 1 J = 1 N \cdot m$$

Force (F)	N
Pressure (p)	Pa, bar, atm, psi
	1 Pa = N/m ² ; 1 atm = 1.01325 bar
	1 bar = 1 x 10 ⁵ N/m ² = 0.1 MPa

Work (W) / Heat (Q)	J
Power (P)	J/s
Sp. internal energy, (u)	J/kg
Sp. enthalpy, (h)	J/kg
Sp. entropy, (s)	J/kg.K
Sp. heat at constant pressure, c _p	J/kg.K
Sp. heat at constant volume, c _v	J/kg.K
Universal gas constant, \bar{R}	8.314 kJ/kgmol.K

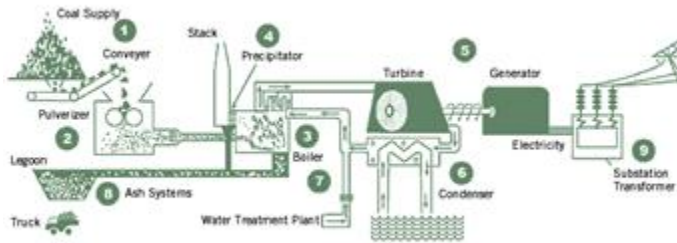
$$\text{Characteristic gas constant, } R_{air} = \frac{\bar{R}}{\mu_{air}} = \frac{8.314}{28.96} = 0.287 \text{ kJ}/(\text{kg} \cdot \text{K})$$

Chapter 1 - Introduction

Why do we need to study thermodynamics?

Knowledge of thermodynamics is required to design any device involving the interchange between heat and work.

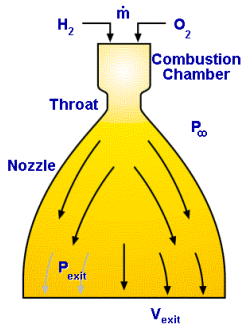
Examples of practical thermodynamic devices:



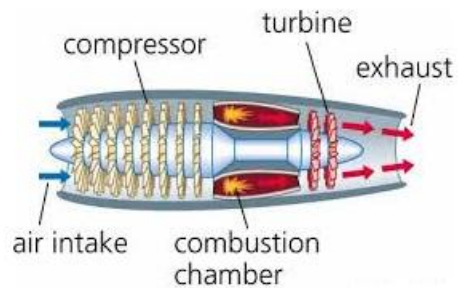
Thermal Power Plant



Air Conditioner



Rocket Nozzle



Turbojet Engine

What is thermodynamics?

- The study of the relationship between work, heat, and energy.
- Deals with the conversion of energy from one form to another.
- Deals with the interaction of a system and its surroundings.

Macroscopic and Microscopic viewpoint of Thermodynamics:

Macroscopic (or Classical Thermodynamics):

- In this approach, a certain quantity of matter is considered, without taking into the account the events occurring at the molecular level.
- In macroscopic thermodynamics, the properties of system are assigned to the system as a whole and that are based on observable, measurable quantities and these effects can be perceived by the human senses.
- Example: A moving car.

Microscopic (or Statistical Thermodynamics):

- From the microscopic view point, matter is composed of large number of small molecules and atoms.
- Microscopic thermodynamics is concerned with the effects of action of many molecules and these effects cannot be perceived by the human senses.
- Example: individual molecules present in the air.

Energy:

Energy is a convenient generic term for “something” which is transferred in the doing of Work.

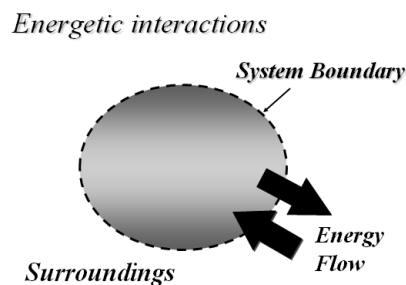
Energy of most interest:

- potential energy (gravity or “spring”)
- kinetic energy
- chemical energy
- internal energy

Above forms can “store” energy, whereas **work** can't. It is only transient manifestation process of transferring energy. **Heat** can't either; it is also only transient manifestation process of transferring energy. So heat and work are other forms of energies in transition.

System: Identifies the subject of the analysis by defining a boundary. Either “a region in space” or “a particular collection of matter” must be treated consistently.

Surroundings: Everything outside the system boundary.



Open System: Fixed volume in space, mass and energy exchange permitted across the system boundary.

Example: jet engine



Closed System: Fixed non-changing mass of fluid within the system, i.e., no mass transfer across the system boundary but can have energy exchange with the surroundings.

Example: piston-cylinder assembly.



Isolated System: a system that does not interact at all with the surroundings, e.g., neither mass nor energy transfer across system boundary.

Properties of a system:

A property of a system is a measurable characteristic of a system that is in equilibrium. Properties may be intensive or extensive.

Intensive property is a physical property of a system that does not depend on the system size or the amount of material in the system.

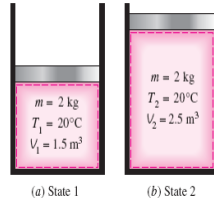
e.g: Temperature, Pressure, and Density

Extensive property of a system does depend on the system size or the amount of material in the system.

e.g: mass, volume, energy, enthalpy

State, Equilibrium and Process:

- ❖ **State** – a set of properties that describes the conditions of a system. Eg. Mass m , Temperature T , volume V .

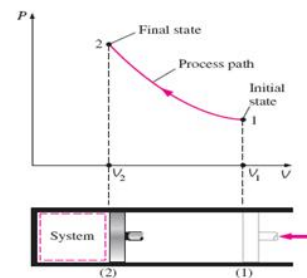


❖ **Change of State:** When system interacts with another system or with surroundings, the system is said to be undergoing change of state.

❖ **Path** – Locus of change of state.

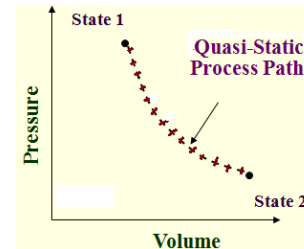
❖ **Process** – If path is specified, process can be defined (based on which property is held constant between initial state and end state).

Process	Property held Constant
isobaric	pressure
isothermal	temperature
isochoric	volume
isentropic	entropy

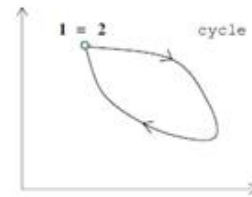


❖ **Thermodynamic Equilibrium** – A system that maintains thermal, mechanical, phase and chemical equilibriums is said to be in thermodynamic equilibrium.

❖ **Quasi-static Process** – When a process is carried out in such a way that the system passes through infinite number of equilibrium states. It is a sufficiently slow process in which, locus of all state points are equilibrium points. A quasi-static process is a hypothetical process and essentially a reversible process.



❖ **Cyclic process** - when a system in a given initial state goes through various processes and finally returns to its initial state, the system has undergone a cyclic process or cycle.



❖ **Reversible process** - it is defined as a process that, once having take place it can be reversed. In doing so, it leaves no change in the system or boundary.

❖ **Irreversible process** - a process that cannot return both the system and surrounding to their original conditions

❖ **Adiabatic process** - a process that has no heat transfer into or out of the system. It can be considered to be perfectly insulated.

❖ **Isentropic process** - a process where the entropy of the fluid remains constant.

- ❖ **Polytropic process** - when a gas undergoes a reversible process in which there is heat transfer. It is represented as $PV^n = \text{constant}$.
- ❖ **Throttling process** - a process in which there is no change in enthalpy, no work is done and the process is adiabatic.

Property Definitions

In order to speak of an intrinsic property “at a point” we must treat matter as a continuum, i.e., matter is distributed continuously in space

- In classical thermodynamics a point represents the smallest volume ‘V’ for which matter can be considered a continuum.
- The value of the property represents an average over this volume ‘V’.

At any instant the *density*, ρ , at a point is defined as

$$\rho = \lim_{V \rightarrow V'} \left(\frac{M}{V} \right) \quad \text{unit: kg/m}^3$$

Mass, M , of the system with volume, V , is

$$\rho = \frac{dM}{dV} \rightarrow M = \int dM = \int_V \rho dV$$

Note: if ρ is uniform over the volume $M = \rho V$

Specific volume, v , defined as

$$v = \frac{1}{\rho} \quad \text{unit: m}^3/\text{kg}$$

The *pressure*, P , at a point is defined as

$$P = \lim_{A \rightarrow A'} \left(\frac{F}{A} \right)$$

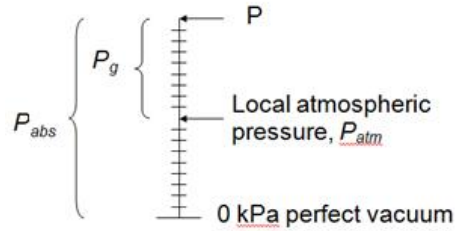
units: $1 \text{ Pa} = 1 \text{ N/m}^2$

1 standard atmosphere = 101,325 Pa

1 bar = 100,000 Pa = 100 kPa = 0.1 MPa

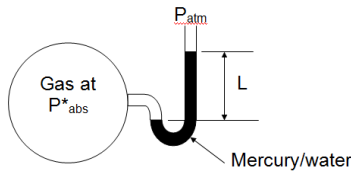
Absolute pressure, P_{abs} , measured relative to a perfect vacuum

Gauge pressure, P_g , measured relative to the local atmospheric pressure, P_{atm} .



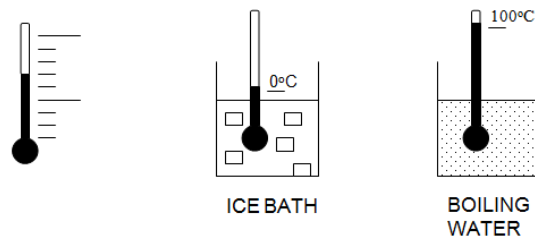
Note: $P_g = P_{abs} - P_{atm}$

Gauge pressure measurement via a manometer



$$P_g = P_{abs} - P_{atm} = \rho g L \quad \text{where, } g = 9.81 \text{ m/s}^2$$

Temperature, T , in units of degrees celcius, $^{\circ}\text{C}$, is a measure of “hotness” relative to the freezing and boiling point of water. A thermometer is based on the thermal expansion of mercury



Temperature in units of degrees kelvin, K , is measured relative to this absolute zero temperature, so $0 \text{ K} = -273^{\circ}\text{C}$; in general, $T \text{ in K} = T \text{ in } ^{\circ}\text{C} + 273.13$.

❖ **Zeroth Law:** “If two bodies are separately in thermal equilibrium with a third body, then they must be in thermal equilibrium with each other.” (Same temperature!).

❖ **First Law and Corollaries**

Work: If Boundary of System moved against a force, Work is done on or by System
(Note: Work is transient, so not a Property)

Heat: Heat is energy transferred from one body to another due to temperature difference
(Note: Heat is transient, so not a property)

❖ **First Law of Thermodynamics:**

When any closed system is undergoing a cycle, the net work delivered by the system is proportional to the net heat transferred to the system,

$$\begin{aligned} \text{i.e., } (\sum Q)_{\text{cycle}} &\propto \sum W_{\text{cycle}} \\ \sum Q &= J \sum W \\ dQ - JdW &= 0 \end{aligned}$$

Where J is Joule's of Mechanical equivalent of Heat
With same units for Q & W; Net heat transfer = Net work.

$$Q_{\text{net}} = W_{\text{net}}$$

❖ **First Law corollary 1:**

There exists a property of a closed system such that a change in its value is equal to the difference between the heat supplied and the workdone during any change of state.

Proof:

Let us consider 1-A-2 & 2-B-1 two processes constituting a cycle, 1-2-1.

From FLTD,

$$(dQ)_A + (dQ)_B = dW_A + dW_B$$

Simply,

$$Q_A + Q_B = W_A + W_B$$

$$Q_A - W_A = -(Q_B - W_B) \text{-----(1)}$$

Let us imagine, the cycle 1-2-1 is completed though the process C instead of B.

Applying FLTD for the cycle 1-2-1,

$$\left(\sum Q_{\text{cycle}} \right) = \left(\sum W_{\text{cycle}} \right)$$

$$Q_A + Q_C = W_A + W_C$$

$$Q_A - W_A = -(Q_C - W_C) \text{-----(2)}$$

Comparing equation 1 & 2

$$\boxed{(Q_B - W_B) = (Q_C - W_C)}$$

Change (Q-W) is same for the process B or C, it is independent of path it followed, so it is a point function and a property. Thus based on FLTD, (dQ - dW) is a property, we call it as internal energy (in kJ) or ('u' - specific internal energy in kJ/kg)

$$\therefore Q_{1-2} - W_{1-2} = U_{1-2} = m(u_2 - u_1)$$

$$dQ - dW = dU \Rightarrow \boxed{dQ = du + dW}$$

❖ **First Law corollary 2:**

If a system is isolated from its surroundings (i.e., $Q = 0$; $W = 0$) then its internal energy remains unchanged

$$du = 0; \quad u = \text{const.}$$

❖ **Work Done in Vacuum**

Rupture of membrane allows the gas to expand into vacuum. The system boundary moves but no force resisting the movement

$$W_{1-2} = 0 \quad (\text{OR}) \quad dW = 0$$

❖ **Perpetual Motion Machine of first kind (PMMI)**

A machine which produces work without absorbing energy from the surroundings is not possible. It must be true if First Law is true.

❖ **Energy Transport by Heat and Work and the Classical Sign Convention**

Energy may cross the boundary of a closed system only by heat or work. Energy transfer across a system boundary due solely to the temperature difference between a system and its surroundings is called heat.

Heat and work are energy transport mechanisms between a system and its surroundings. The similarities between heat and work are as follows:

- a) Both are recognized at the boundaries of a system as they cross the boundaries. They are both boundary phenomena.
- b) Systems possess energy, but not heat or work.
- c) Both are associated with a process, not a state. Unlike properties, heat or work has no meaning at a state.
- d) Both are path functions (i.e., their magnitudes depends on the path followed during a process as well as the end states.

Since heat and work are path dependent functions, they have inexact differentials designated by the symbol, δ . The differentials of heat and work are expressed as δQ and δW . The integral of the differentials of heat and work over the process path gives the amount of heat or work transfer that occurred at the system boundary during a process.

$$\int_1^2 \delta Q = Q_{12} \quad (\text{not } \Delta Q)$$

$$\int_1^2 \delta W = W_{12} \quad (\text{not } \Delta W)$$

That is, the total heat transfer or work is obtained by following the process path and adding the differential amounts of heat (δQ) or work (δW) along the way. The integrals of δQ and δW are not $Q_2 - Q_1$ and $W_2 - W_1$, respectively, which are meaningless since both heat and work are not properties and systems do not possess heat or work at a state.

Work has the units of energy and is defined as force times displacement or Newton times meter or joule. Work per unit mass of a system is measured in kJ/kg.

❖ Common Types of Mechanical Work Energy

- Shaft Work
- Spring Work
- Work done of Elastic Solid Bars
- Work Associated with the Stretching of a Liquid Film
- Work Done to Raise or to Accelerate a Body

Net Work Done By a System

The net work done by a system may be in two forms: other work and boundary work. First, work may cross a system boundary in the form of a rotating shaft work, electrical work or other the work forms listed above. We will call these work forms “other” work, that is, work not associated with a moving boundary. Second, the system may do work on its surroundings because of moving boundaries due to expansion or compression processes that a fluid may experience in a piston-cylinder device.

$$W_{net} = \left(\sum W_{out} - \sum W_{in} \right)_{other} + W_b$$

Here, W_{out} and W_{in} are the magnitudes of the other forms of work crossing the boundary. W_b is the work due to the moving boundary as would occur when a gas contained in a piston cylinder device expands and does work to move the piston. The boundary work will be positive or negative depending upon the process.

$$W_{net} = (W_{net})_{other} + W_b$$

❖ Introduction to the Basic Heat Transfer Mechanisms

Heat transfer is energy in transition due to a temperature difference. The three modes of heat transfer are conduction, convection, and radiation.

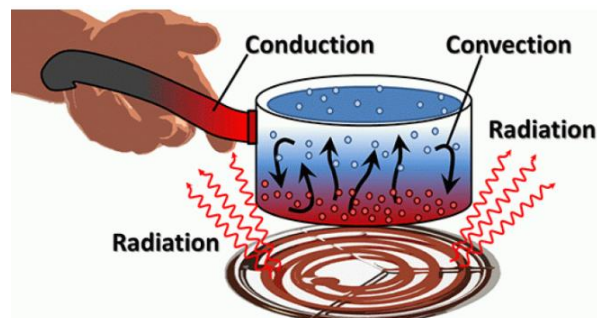


Figure: Illustration of Modes of Heat Transfer

Conduction: It is the mode of heat transfer particularly in solids. In this mode of heat transfer, the heat transfers from one atom to its neighbouring atom through molecular vibrations.

Rate of heat transfer by conduction, \dot{Q}_{cond} is governed by Fourier Law of heat conduction:

$$\dot{Q}_{cond} = -k A \frac{dT}{dx},$$

Where k - thermal conductivity of the chosen material (W/mK).

A – Area normal to the heat flow (m²)

dT/dx – Temperature gradient in the direction of heat flow (°C/m)

Convection: This mode of heat transfer particularly occurs in fluids in motion. That is in both liquids and gases that are in motion. This mode of heat transfer occurs due to the transfer of energy through the bulk mass.

In detail whenever there is a temperature difference in a fluid, density difference occurs and motion of fluid starts as lower density fluid attempts to reach the top of the fluid. During this motion mass and energy transfer occurs thus heat transfer takes place.

Rate of heat transfer by convection, \dot{Q}_{conv} is governed by Newton's Law of cooling, $\dot{Q}_{conv} = h A dT$, where h = Convective heat transfer coefficient (W/m²K).

Radiation: In this mode, heat transfer takes place via electromagnetic waves. Electromagnetic waves transport energy like other waves and travel at the speed of light. This is the mode by which we receive solar energy from the sun.

Rate of heat transfer by Radiation, \dot{Q}_{rad} is governed by Stefan-Boltzmann law, $\dot{Q}_{rad} = \sigma A (T_s^4 - T_{surr}^4)$, where σ – Stefan-Boltzmann constant, 5.67×10^{-8} W/m²K⁴

❖ **Ideal gas equation:**

Gas Law Formula		
Gas Law	Formula	Description
Boyle's Law	$P_1V_1 = P_2V_2$	At constant T, as pressure increases, volume decreases.
Charles' Law	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	At constant P, as volume increases, temperature increases.
Gay-Lussac's Law	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$	At constant V, as pressure increases, temperature increases.

Equation of state is defined from Avogadro's law, which states that, equal volumes of all gases, at the same temperature and pressure, have the same number of molecules".

For a given mass of an ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant.

$$p\bar{v} = \bar{R}T$$

if $p = 760 \text{ mm of Hg} = 1.01325 \text{ bar}$; $T = 273.15 \text{ K}$, it occupies $22.4 \text{ m}^3/\text{kgmol}$

$$\text{Universal Gas Constant, } \bar{R} = \frac{p\bar{v}}{T} = \frac{1.01325 \times 10^5 \times 22.4}{273.15} = 8.314 \frac{\text{kJ}}{\text{kgmol.K}}$$

We know that $\bar{v} = \frac{V}{n}$ where $n = \text{number of moles}$

So gas equation can be written as,

$$pV = n\bar{R}T ; \quad \text{where } n = \frac{m}{\mu}$$

m is the mass of the gas and μ is the molecular weight of the gas.

$$pV = \frac{m}{\mu} \bar{R}T = m \left(\frac{\bar{R}}{\mu} \right) T$$

$\Rightarrow pV = mRT$ where R is the characteristic gas constant.

❖ **Enthalpy:**

It is a composite property. It is defined or sum of internal energy and flow energy.

$$\text{Enthalpy} \quad : \quad H = U + PV$$

$$\text{Specific Enthalpy} \quad : \quad h = u + pv$$

From FLTD,

$$dQ = du + dW$$

$$= du + pdV$$

$$= d(u + pV)$$

$$(dQ)_{p=c} = dh$$

$$\left\{ \begin{array}{l} pdv = d(pv) \\ \text{for constant pressure process} \end{array} \right.$$

For constant pressure process $dQ = (h_2 - h_1)$

❖ **Specific heat at Constant volume : C_V :**

Defined as the rate of change with temperature of the specific internal energy of the system when volume is held constant.

$$C_V = \left(\frac{\partial u}{\partial T} \right)_V$$

From FLTD,

$$dQ = du + dW$$

$$dQ = du$$

$$| \quad dW = 0, \text{ for constant volume process}$$

$$\therefore \text{ Heat transfer for constant volume process } \{ dQ_{V=C} = u_2 - u_1$$

So for a small temperature change dT ,

$$\left[\frac{dQ}{dT} \right]_V = \left[\frac{du}{dT} \right]_V$$

But $\left[\frac{dQ}{dT} \right]_V$ is specific heat and $\left[\frac{dQ}{dT} \right]_V$ is specific heat at constant volume- C_V

C_V is a property because is only expressed in terms of other properties: *expressed in $\frac{kJ}{kg K}$*
thus it must be a property.

For perfect gas,

$$\left[\frac{dQ}{dT} \right]_V = C_V; \quad \rightarrow [dQ]_V = C_V(dT)$$

$$\text{Heat transfer for constant volume process } [dQ]_V = C_V(T_2 - T_1)$$

❖ Specific heat at constant pressure: (C_P) :

Defined as the state of change with temperature of the specific enthalpy of the system when the pressure is held constant

$$\text{i.e., } C_P = \left[\frac{dh}{dT} \right]_P$$

From FLTD,

$$\begin{aligned} dQ &= du + pdv \\ &= d(u + PV) \\ [dQ]_P &= dh \end{aligned}$$

For small temperature change dT ,

$$\left[\frac{dQ}{dT} \right]_P = \left[\frac{dh}{dT} \right]_P$$

$\left[\frac{dQ}{dT} \right]_P$ is specific heat at constant pressure.

$$\therefore C_P = \left[\frac{dQ}{dT} \right]_P$$

$$[dQ]_P = C_P Dt$$

Heat transfer for constant pressure process $\{[dQ]_P = C_P(T_2 - T_1)\}$

$$[dQ]_P = dh = C_P(T_2 - T_1)$$

$$h_2 - h_1 = C_P(T_2 - T_1)$$

❖ Ratio of specific heats: $\left(\frac{C_P}{C_V}\right)$

$$\gamma = \left(\frac{C_P}{C_V}\right)$$

❖ Relationships between C_P , C_V , & R

From FLTD,

$$dQ = du + pdV$$

$$h = u + PV$$

$$dh = du + pdv + vdp$$

We know that for perfect gas $PV = RT$

For constant pressure process,

$$dh = du + d(PV) + 0$$

$$dh = du + d(RT)$$

$$dh = du + RdT$$

÷ by dT

$$\frac{dh}{dT} = \frac{du}{dT} + R$$

$$C_P = C_V + R \quad \rightarrow \quad C_P - C_V = R$$

Where $R \rightarrow$ characteristics gas constant

$$R = \frac{\bar{R}}{\mu} \quad \text{where } \bar{R} \text{ is universal gas constant}$$

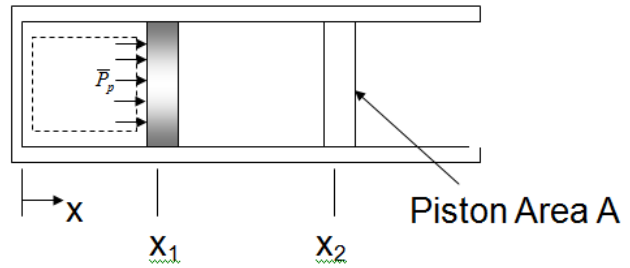
$$\bar{R} = 8.314 \text{ KJ/Kmol.K}$$

$\mu =$ Molecular weight of a gas.

$$R_{air} = \frac{\bar{R}}{\mu_{air}} = \frac{8.314}{28.97} \quad \rightarrow \quad R_{air} = 0.287 \text{ kJ/kg.K}$$

❖ **Thermodynamic work or pdv work:**

Let us consider a piston-cylinder arrangement. A gas is enclosed in the cylinder at Position 1, the pressure, temperature and volume of gas in the cylinder is P, T & V , if the gas is allowed to expand to an elemental distance dl , the force acting on the surface of the piston is,



The pressure forces acting on the piston, $F = [P * A]$

Workdone by the expanding gas in the piston for elemental distance dl is,
 $dW = PdV$

Workdone by the gas from state 1 to 2, $W_{1-2} = \int_1^2 PdV$

Thus, the area under the process curve on a p-V diagram is equal to the workdone during a quasi-static process of a closed system.

❖ **Property Relation**

Properties have unique values for any (equilibrium) state. Properties are state functions and independent of path or process.

$$\int_1^2 dP = P_2 - P_1 \quad \text{is exact differential.}$$

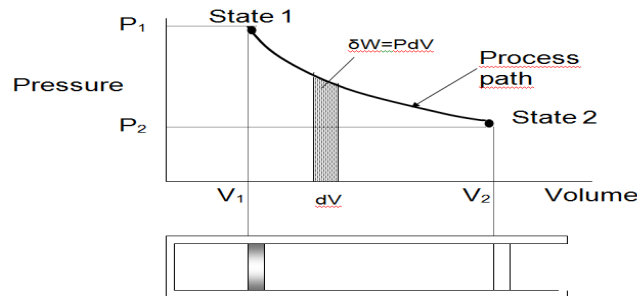
$$\text{Similarly; } \int_1^2 dV = V_2 - V_1$$

but work, not a property, because it is not independent of process.

$$\int_1^2 dW \neq W_2 - W_1$$

$$\int_1^2 dW = W_{1-2} \text{ (OR) } W_{2-1}; \quad \text{It is an inexact differential.}$$

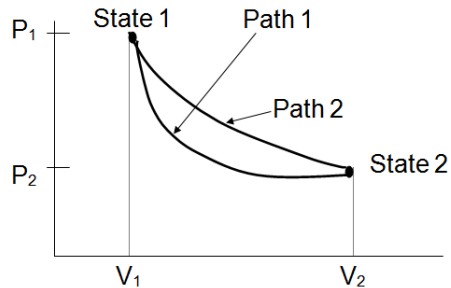
❖ **Work is a path function:**



$$\delta W = PdV \quad \text{shaded area}$$

$$W_{1 \rightarrow 2} = \int_1^2 \delta W = \int_{V_1}^{V_2} PdV \quad \text{total area under curve}$$

Consider two processes with the same start and end state



Since the area under each curve is different the amount of work done for each path is different.

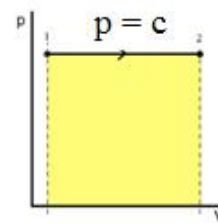
$$\left(\int_{V_1}^{V_2} PdV \right)_{\text{path 1}} \neq \left(\int_{V_1}^{V_2} PdV \right)_{\text{path 2}}$$

Work done depends on the path taken and not just the value of the end states. Work is not a property!

❖ **PdV work for different thermodynamics processes:**

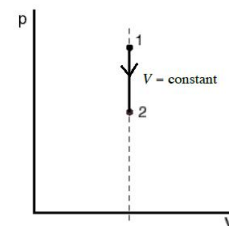
✓ **Constant pressure process (Isobaric process) :**

$$\begin{aligned} W_{1-2} &= \int_1^2 PdV \\ &= P \int_1^2 dV \\ \boxed{(W_{1-2})} &= P(V_2 - V_1) \end{aligned}$$



Constant Volume process (Isochoric Process)

$$\begin{aligned} (W_{1-2})_{V=c} &= \int_1^2 PdV \\ (W_{1-2})_{V=c} &= \mathbf{0} \\ \text{Because } V_1 &= V_2; \quad V = \text{Const} \quad dV = 0 \end{aligned}$$



✓ **Constant Temperature Process (Isothermal Process)**

$$W_{1-2} = \int_1^2 P dV$$

We know that gas equation, $PV = RT$

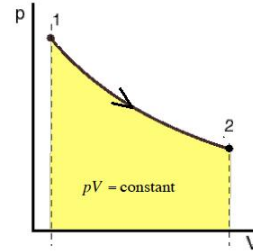
When, $T = C$

$$\therefore P_1V_1 = P_2V_2 = PV = C$$

$$P = \frac{P_1V_1}{V} \dots \dots \dots (1)$$

$$(W_{1-2})_{\text{isothermal}} = \int_1^2 \frac{P_1V_1}{V} dV = P_1V_1 \int_1^2 \frac{dV}{V}$$

$$(W_{1-2})_{\text{isothermal}} = P_1V_1 \ln \frac{V_2}{V_1}$$



✓ **Polytropic Process**

$$P_1V_1^n = P_2V_2^n = PV^n = \text{Const}$$

$$\Rightarrow P = \frac{P_1V_1^n}{V^n}$$

$$(W_{1-2})_{\text{poly}} = \int_1^2 P dV = P_1V_1^n \int_1^2 \frac{dV}{V^n}$$

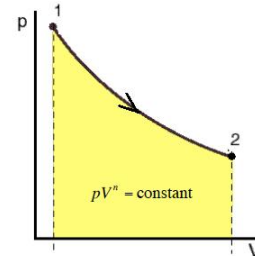
$$\Rightarrow (W_{1-2})_{\text{poly}} = P_1V_1^n \left[\frac{V^{-n+1}}{1-n} \right]$$

$$= \frac{P_1V_1^n V_2^{1-n} - P_1V_1^n V_1^{1-n}}{1-n}$$

$$= \frac{P_2V_2^n V_2^{1-n} - P_1V_1^n V_1^{1-n}}{1-n}$$

$$= \frac{P_2V_2 - P_1V_1}{1-n}$$

$$(W_{1-2})_{\text{poly}} = \frac{P_1V_1 - P_2V_2}{n-1}$$



❖ **Property relations for different thermodynamic processes:**

✓ **Constant pressure process (Isobaric process, $p_1 = p_2$)**

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \rightarrow \quad \frac{T_2}{T_1} = \frac{v_2}{v_1}$$

✓ **Constant volume process (Isochoric process, $v_1 = v_2$)**

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \rightarrow \quad \frac{T_2}{T_1} = \frac{p_2}{p_1}$$

✓ **Constant temperature process (Isothermal process, $T_1 = T_2$)**

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \rightarrow \quad \frac{p_2}{p_1} = \frac{v_1}{v_2}$$

✓ **For polytropic process ($p v^n = c$)**

$$p_1 v_1^n = p_2 v_2^n \quad \rightarrow \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \rightarrow \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right) \left(\frac{v_2}{v_1}\right) \quad \rightarrow \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^n \left(\frac{v_2}{v_1}\right)$$

$$\rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} \quad \rightarrow \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right) \left(\frac{v_2}{v_1}\right) \quad \rightarrow \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right) \left(\frac{p_1}{p_2}\right)^{1/n}$$

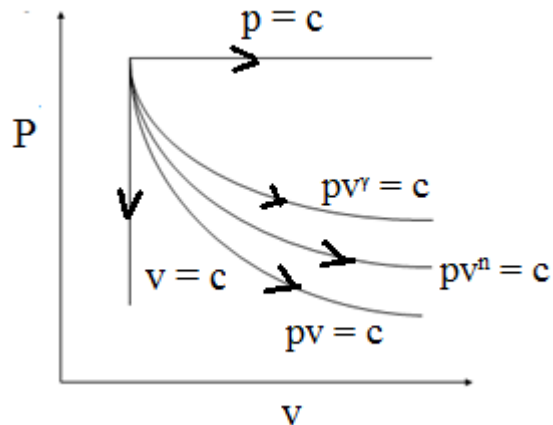
$$\rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right) \left(\frac{p_2}{p_1}\right)^{-1/n}$$

$$\rightarrow \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

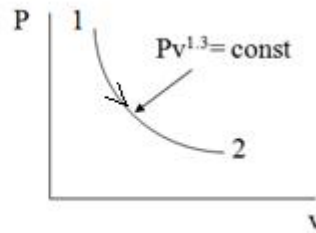
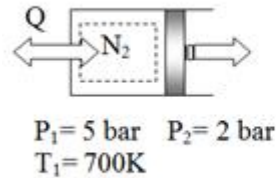
Summary of Thermodynamic Processes

Process	Constant Pressure (Isobaric)	Constant Volume (Isochoric)	Constant Temp. (Isothermal)	Rev. Adiabatic (Isentropic)	Polytropic
Index (n)	n = 0	n = ∞	n = 1	n = γ	n = n
p v T Relations	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$ Charles's Law	$\frac{T_2}{T_1} = \frac{p_1}{p_2}$	$p_1 v_1 = p_2 v_2$ Boyle's Law	$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$	$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$
Specific heat C	c_p	c_v	∞	0	$c_n = c_v \left(\frac{\gamma-n}{1-n}\right)$
Heat added, Q	$c_p(T_2 - T_1)$	$c_v(T_2 - T_1)$	$p_1 v_1 \ln \frac{v_2}{v_1}$	0	$\left(\frac{\gamma-n}{\gamma-1}\right) * WD$
Work Done, W	$p(v_2 - v_1)$	0	$p_1 v_1 \ln \frac{v_2}{v_1}$	$\frac{p_1 v_1 - p_2 v_2}{\gamma-1}$	$\frac{p_1 v_1 - p_2 v_2}{n-1}$
Entropy Change S ₂ - S ₁	$c_p \ln \frac{T_2}{T_1}$	$c_v \ln \frac{T_2}{T_1}$	$R \ln \frac{v_2}{v_1}$	0	$c_v \left(\frac{n-\gamma}{n-1}\right) * \ln \left(\frac{T_2}{T_1}\right)$

Different Thermodynamic Processes:



Problem #1.1: A piston-cylinder assembly contains 1 kg of nitrogen gas (N_2). The gas expands from an initial state where $T_1 = 700\text{K}$ and $P_1 = 5\text{ bar}$ to a final state where $P_2 = 2\text{ bar}$. During the process the pressure and specific volume are related by $Pv^{1.3} = \text{const}$. Assuming ideal gas behaviour and neglecting KE and PE effects, determine the heat transfer during the process, in kJ.



From FLTD,

$$\Delta U = Q - W$$

$$Q = \Delta U + W$$

$$W = \int_1^2 p dV = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

Recall, for an ideal gas $PV = mRT$

$$\int_1^2 p dV = \frac{mR(T_1 - T_2)}{n - 1} \quad \text{need } T_2$$

For polytropic process,

$$\rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{n-1}$$

$$T_2 = \left(\frac{2\text{ bar}}{5\text{ bar}} \right)^{\frac{0.3}{1.3}} (700\text{K}) = 567\text{K}$$

$$W = \frac{mR(T_1 - T_2)}{n - 1}$$

$$W = \frac{1(0.287)(700 - 567)}{1.3 - 1} = +132\text{kJ}$$

Note: work is positive \rightarrow work done by the system

The average value $\tilde{c}_v = 0.786 \text{ kJ/kg}\cdot\text{K}$ is assumed for air.

$$\begin{aligned}\Delta U &= m(u_2 - u_1) = mc_v(T_2 - T_1) \\ &= 1 \text{ kg} (0.786 \text{ kJ/kg}\cdot\text{K})(567 - 700)\text{K} = -104.5 \text{ kJ}\end{aligned}$$

$$Q = \Delta U + W \Rightarrow Q = (-104.5 \text{ kJ}) + (132 \text{ kJ}) = +27.5 \text{ kJ}$$

Note: heat transfer is positive \rightarrow So, heat is transferred into the system.

Problem #1.2: 0.1 m^3 of an ideal gas at 300 K and 1 bar is compressed adiabatically to 8 bar . It is then cooled at constant volume and further expanded isothermally so as to reach the condition from where it started. Calculate a) pressure at the end of constant volume cooling, b) change in internal energy during constant volume process, c) net work done and heat transferred during the cycle. Assume $c_p = 14.3 \text{ kJ/kg}\cdot\text{K}$ and $c_v = 10.2 \text{ kJ/kg}\cdot\text{K}$.

Solution:

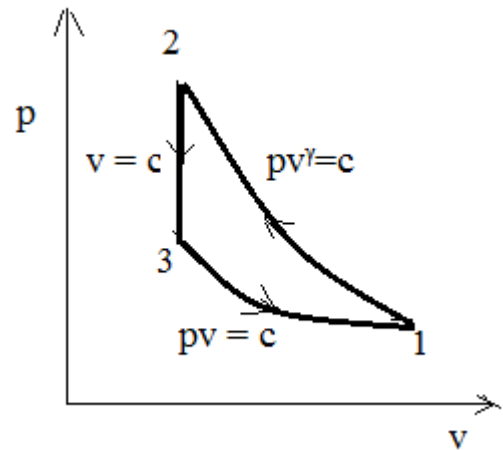
$$\gamma = \frac{c_p}{c_v} = \frac{14.3}{10.2} = 1.4$$

For Process 1-2 (Adiabatic Compression):

$$p_1 V_1^\gamma = p_2 V_2^\gamma \rightarrow V_2 = 0.0227 \text{ m}^3$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_2 = 544.5 \text{ K}$$

$$W_{1-2} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \rightarrow W_{1-2} = -20.27 \text{ kJ}$$



For Process 2-3 (Constant Volume Cooling):

$$\text{Change in internal energy, } u_3 - u_2 = m c_v(T_3 - T_2) = -20.2 \text{ kJ}$$

$$\text{Work done, } W_{2-3} = 0$$

For Process 3-1 (Isothermal Expansion):

$$W_{3-1} = p_3 v_3 \ln\left(\frac{v_1}{v_3}\right) = p_1 v_1 \ln\left(\frac{p_3}{p_1}\right); \text{ Because, } p_3 v_3 = p_1 v_1 \text{ for isothermal Process}$$

$$\text{Therefore, } W_{3-1} = 14.82 \text{ kJ}$$

$$\text{Net work done, } W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1} = -5.45 \text{ kJ}$$

-ve sign indicates work has been done on the system.

From FLTD, for a cyclic process;

$$(\sum W)_{\text{cycle}} = (\sum Q)_{\text{cycle}} \rightarrow Q_{\text{net}} = -5.45 \text{ kJ} \quad \text{-ve sign indicates heat is lost from the system.}$$

Problem #1.3: A fluid at 4.15 bar is expanded reversibly according to a law $PV = \text{constant}$ to a pressure of 1.15 bar until it has a specific volume of $0.12 \text{ m}^3/\text{kg}$. It is then cooled reversibly at a constant pressure, then is cooled at constant volume until the pressure is 0.62 bar; and is then allowed to compress reversibly according to a law $PV^n = \text{constant}$ back to the initial conditions. The work done in the constant pressure is 0.525 kJ, and the mass of fluid present is 0.22 kg. Calculate the value of n in the fourth process, the net work of the cycle and sketch the cycle on a P-V diagram.

Solution:

Given Data:

$$p_1 = 4.15 \text{ bar}; p_2 = 1.15 \text{ bar}; v_2 = 0.12 \text{ m}^3/\text{kg}$$

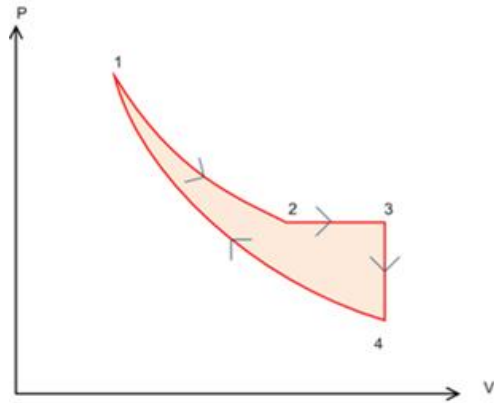
$$p_4 = 0.62 \text{ bar}; W_{(p=c)} = 0.525 \text{ kJ}; m = 0.22 \text{ kg}$$

Process 1-2 (Isothermal):

$$p_1 v_1 = p_2 v_2$$

$$v_1 = \left(\frac{p_2}{p_1}\right) v_2 = \mathbf{0.00732 \text{ m}^3}$$

$$W_{1-2} = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = \mathbf{3.895 \text{ kJ}}$$



Process 2-3 (Constant Pressure):

$$W_{2-3} = p_2 (v_3 - v_2) = \mathbf{0.525 \text{ kJ}}$$

$$v_3 = \frac{0.525}{1.15} + 0.0264 = \mathbf{0.03097 \text{ m}^3}$$

Process 3-4 (Constant Volume):

$$W_{3-4} = 0$$

Process 4-1 (Polytropic):

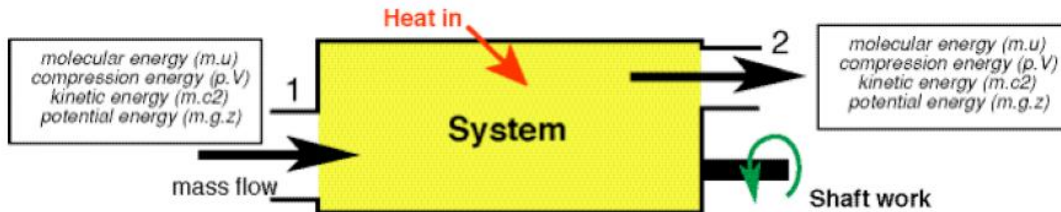
$$\frac{p_4}{p_1} = \left(\frac{v_1}{v_4}\right)^n = \frac{62}{415} = \left(\frac{0.00732}{0.03097}\right)^n \rightarrow \ln(0.1494) = n \cdot \ln(0.2364) \rightarrow \mathbf{n = 1.3182}$$

$$W_{4-1} = \frac{p_4 v_4 - p_1 v_1}{n-1} = \mathbf{-3.5124 \text{ kJ}}$$

$$W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1} \rightarrow \mathbf{W_{\text{net}} = 0.9076 \text{ kJ}}$$

❖ STEADY FLOW PROCESS

Many engineering devices such as nozzles, compressors and turbines operate a steady state conditions i.e., properties are not changing with time. Let us consider an open system where system boundary is fixed. The confined region where attention is focused can be selected as a control volume.



Applying the principle of conservation of energy for a general steady flow system as shown in fig.

$$\begin{aligned}
 & [\text{total energy crossing the boundary as heat and work per unit time}] \\
 & = [\text{Total energy flowing net of control volume with mass per unit time}] \\
 & - [\text{Total energy having into control volume with mass per unit time}]
 \end{aligned}$$

The flowing mass is having the following energies at inlet and exit i.e., molecular internal energy (mu), flow energy (pv), kinetic energy ($mc^2/2$), and potential energy (mgz)
(The total energy of flowing

$$\begin{aligned}
 \text{Fluid per unit mass at entry)} \quad \Sigma_{in} \text{ energy} &= \left(u_1 + \frac{c_1^2}{2} + z_1 g \right) + P_1 V_1 \\
 &= \left(h_1 + \frac{c_1^2}{2} + z_1 g \right) \text{ in kJ/kg}
 \end{aligned}$$

$$\text{At exit,} \quad \Sigma_{out} \text{ energy} = \left(h_2 + \frac{c_2^2}{2} + z_2 g \right) \text{ in kJ/kg}$$

Applying energy principle

$$\dot{m}_1 \left(h_1 + \frac{c_1^2}{2} + z_1 g \right) + Q_{net} = \dot{m}_2 \left(h_2 + \frac{c_2^2}{2} + z_2 g \right) + W_{net} \quad \dots\dots (2)$$

$$(Q_{net} - W_{net}) = \dot{m} \left[(h_2 - h_1) + \left(\frac{c_2^2 - c_1^2}{2} \right) + (z_2 - z_1)g \right] \quad \dots\dots (3)$$

Where $\dot{m}_1 = \dot{m}_2$ law of conservation of mass.

Eqn (3) is the general form of steady flow energy equation where Q & W and rate of heat and work transfer respectively in $\frac{KJ}{s}$.

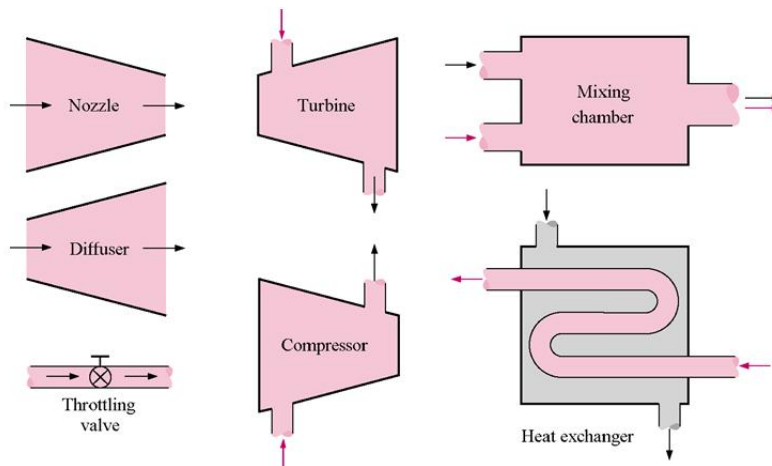
Dividing the eqn (3) by \dot{m} we obtain

$$\frac{Q_{net}}{\dot{m}} - \frac{W_{net}}{\dot{m}} = (h_2 - h_1) + \frac{c_2^2 - c_1^2}{2} + (z_2 - z_1)g$$

$$q_{net} - w_{net} = (h_2 - h_1) + \frac{c_2^2 - c_1^2}{2} + (z_2 - z_1)g$$

Where the subscript 1 & 2 stands for inlet and outlet conditions.

❖ **STEADY FLOW DEVICES:**



Applying steady flow energy equation for devices like nozzle, compressor and turbine, the equation can be simplified as follows;

A nozzle is a device which increases the velocity of a fluid at the expense of pressure,

Applying SFEE for nozzle

$$q - w = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + (z_2 - z_1)g$$

In nozzle flow, the rate of heat transfer between the fluid and the surrounding is usually insignificant because of the fact that the fluid will be flowing at high velocity and they pass through a nozzle quickly without any significant heat transfer taking place ($q = 0$); The shaft work ($W = 0$) for a nozzle flow PE change is also negligible.

$$0 = h_2 - h_1 + \frac{C_2^2 - C_1^2}{2} = C_2^2 = 2(h_1 - h_2) \quad \text{because, } C_2 \gg C_1$$

$$V_2 = \sqrt{2(h_1 - h_2)} \quad \text{in m/s where } h_1 \& h_2 \text{ in J/kg}$$

Applying SFE principle to flow through compressor

$$\Delta PE = 0; \Delta KE = 0 \& W \neq 0$$

There is shaft work involved. Compressor is driven by external power; work is done on the system. $q = 0$ (heat rate is small compared to the shaft work). Therefore, *SFEE becomes*,

$$-W = h_1 - h_2;$$

$$W = -(h_1 - h_2) \quad \text{[-ve sign indicates, work is done on the system for compressor]}$$

For turbine, all conditions are similar to the compressor except that work done is positive

$$W = (h_1 - h_2) \quad \text{in kJ/kg}$$

For throttling valve, there is a significant pressure drop in the fluid where it is passed through a restricted passage. No work and heat transfer considered. ΔPE & ΔKE is also insignificant. Therefore, the SFEE becomes $\boxed{h_1 = h_2}$

Problem # 1.4: Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s . The inlet area of the diffuser is 0.4 m^2 . The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine, a) The mass flow rate of the air, b) The temperature of the air leaving the diffuser.

Given Data:

$$\begin{aligned} P_1 &= 80\text{ kPa} \\ C_1 &= 200\text{ m/s} \\ T_1 &= 10^\circ\text{C} = 283\text{ K} \\ A_1 &= 0.4\text{ m}^2 \\ V_2 &= 0 \\ \dot{m} &=? \\ T_2 &=? \end{aligned}$$

Solution:

Simplified SFEE for diffuser flow is

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad [q_{out} = W_{net} = 0]$$

Enthalpy at state 1, $h_1 = C_p T_1 = 1.005 * 283 = 284.42\text{ kJ/kg}$

$$\begin{aligned} \text{Mass flow rate, } \dot{m} &= \rho_1 A_1 C_1 \\ &= 0.9852 * 200 * 0.4 \end{aligned}$$

$$\dot{m} = 78.8\text{ kg/s}$$

From gas equation $P_1 V_1 = m R T_1$

$$P_1 V_1 = R T_1$$

$$V_1 = \frac{P_1}{R T_1} = 1.015 \frac{\text{m}^3}{\text{kg}}$$

$$\rho_1 = \frac{1}{V_1} = 0.9852\text{ kg/m}^3$$

Enthalpy at state 2, can be obtained by,

Substituting h_1 & C_1 values in SFEE,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$h_2 = h_1 + \frac{C_1^2}{2}$$

$$= (284.42 * 10^3) + \frac{200^2}{2}$$

$$\boxed{h_2 = 304.42\text{ kJ/kg}}$$

$$h_2 = C_p T_2$$

$$T_2 = \frac{304.42}{1.005} = 302.92\text{ K}$$

Problem # 1.5: Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The Mass flow rate of the air is 0.02 kg/s and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary process input to the compressor.

Given Data:

$$\begin{aligned}
 P_1 &= 100 \text{ kPa} \\
 T_1 &= 280 \text{ K} \\
 \dot{m} &= 0.02 \text{ kg/s} \\
 P_2 &= 600 \text{ kPa} \\
 T_2 &= 400 \text{ K} \\
 q &= -16 \text{ kJ/kg} \\
 \Delta KE &= \Delta PE = 0
 \end{aligned}$$

SOLUTION:

Simplified energy balance equation is

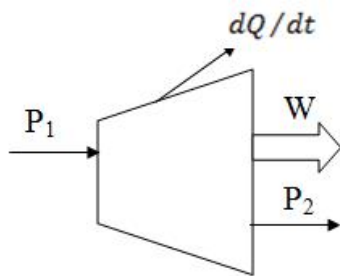
$$\begin{aligned}
 Q_{out} - W_{in} &= \dot{m}(h_2 - h_1) \\
 W_{in} &= Q_{out} - \dot{m}(h_2 - h_1) \\
 W_{in} &= \dot{m}q_{out} - \dot{m}(h_2 - h_1) \\
 &= \dot{m}(q_{out} - (h_2 - h_1)) \\
 h_2 = C_p T_2 &= 400.98 \text{ kJ/kg} ; \quad h_1 = C_p T_1 = 280.13 \text{ kJ/kg} \\
 W_{in} &= 0.02[(-16) - (400.98 - 280.13)]
 \end{aligned}$$

$$\boxed{W_{in} = -2.74 \text{ kW}}$$

Negative (-'ve) sign indicates, work is done on the system.

Problem #1.6: In a gas turbine installation, the gases enter the turbine at the rate of 5 kg/s with a velocity of 50 m/s and enthalpy of 900 kJ/kg and leave the turbine with 150 m/s and enthalpy of 400 kJ/kg. The loss of heat from the gases to the surroundings is 25 kJ/kg. Assume $R = 0.285 \text{ kJ/kgK}$, $c_p = 1.004 \text{ kJ/kgK}$ and inlet conditions to be 100 kPa and 27 °C. Determine the diameter of the inlet pipe.

Solution:



$$\frac{dQ}{dt} = -'ve \text{ (heat is lost to the surroundings)}$$

$$\frac{dQ}{dt} = - \left(25 \frac{\text{kJ}}{\text{kg}} * 5 \frac{\text{kg}}{\text{s}} \right) = 125 \times 10^3 \text{ J/s}$$

$$\dot{m}_1 = \dot{m}_2 = 5 \text{ kg/s}$$

Applying SFEE,

$$\dot{m}_1 \left(h_1 + \frac{c_1^2}{2} + Z_1 g \right) + \cancel{\frac{dQ}{dt}} = \dot{m}_2 \left(h_2 + \frac{c_2^2}{2} + Z_2 g \right) + \cancel{\frac{dW}{dt}}$$

$$\frac{dW}{dt} = 5[(900 \times 10^3 - 400 \times 10^3) + \left(\frac{50^2 - 150^2}{2} \right)] - 125 \times 10^3$$

$$\text{Turbine Work, } \frac{dW}{dt} = W_{1-2} = 2325 \text{ kJ/s}$$

To get Dia of Inlet Pipe:

Volume of gas entering the turbine per sec,

$$\text{From gas equation, } v_1 = \frac{m_1 R T_1}{p_1} = \frac{5 \times 285 \times 300}{100 \times 10^3} = 4.275 \frac{\text{m}^3}{\text{s}}$$

From continuity equation, $\dot{m}_1 v_1 = A_1 \times C_1$

$$\text{Inlet area of the pipe, } A_1 = \frac{\pi D_1^2}{4} = 0.4275 \implies D = 73 \text{ cm}$$

Supplementary Problems for Practice

1. A mass of 0.15 kg of air is initially exists at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle. ***[-4.75 kJ]***

2. 0.078 kg of a carbon monoxide initially exists at 130 kPa and 120°C. The gas is then expanded polytropically to a state of 100 kPa and 100°C. Sketch the P-V diagram for this process. Also determine the value of n (index) and the boundary work done during this process. ***[1.248, 1.855 kJ]***

3. A gas contained within a piston-cylinder assembly undergoes three processes in series: Process 1-2: Compression with $pV=C$ from $p_1 = 1$ bar, $V_1 = 1\text{ m}^3$ to $V_2 = 0.2\text{ m}^3$. Process 2-3: Constant pressure expansion to $V_3 = 1\text{ m}^3$. Process 3-1: Constant volume. Sketch the processes in series on a p-V diagram labeled with pressure and volume values at each numbered state.

4. A gas undergoes a thermodynamic cycle consisting of the following processes: Process 1-2: Constant pressure, $p = 1.4$ bar, $V_1 = 0.028\text{ m}^3$, $W_{1-2} = 10.5$ kJ; Process 2-3: Compression with $pV = \text{constant}$, $U_3 = U_2$; Process 3-1: Constant volume, $U_1 - U_3 = -26.4$ kJ. There are no significant changes in KE and PE. a) Sketch the cycle on a p-V diagram, b) Calculate the net work for the cycle in kJ.

5. A system contains 0.15 m^3 of air pressure of 3.8 bars and 150°C. It is expanded adiabatically till the pressure falls to 1.0 bar. The air is then heated at a constant pressure till its enthalpy increases by 70 kJ. Sketch the process on a P-V diagram and determine the total work done. Use $c_p = 1.005$ kJ/kg.K and $c_v = 0.714$ kJ/kg.K ***[54.9 kJ]***

6. If a gas of volume 6000 cm^3 and at a pressure of 100 kPa is compressed quasi-statically according to $pv^{1.2} = \text{constant}$ until the volume becomes 2000 cm^3 , determine the final pressure and the work transfer. ***[382 kPa, 0.82 kJ]***

7. A certain gas of mass 4 kg is contained within a piston-cylinder assembly. The gas undergoes a process for which $pv^{1.5} = \text{constant}$. The initial state is given by 3 bar, 0.1 m^3 . The change in internal energy of the gas in the process is, $u_2 - u_1 = -4.6$ kJ/kg. Find the net heat transfer for the process when the final volume is 0.2 m^3 . Neglect the changes in KE and PE. ***[-0.8 kJ]***

8. A turbine operates under steady flow conditions, receiving steam at the following state: pressure 1.2 MPa, temperature 188 °C, enthalpy 2785 kJ/kg, velocity 33.3 m/s and elevation 3m. The steam leaves the turbine at the following state: pressure 20kPa, enthalpy 2512 kJ/kg, velocity 100 m/s and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/s. If the rate of steam flow through the turbine is 0.42 kg/s, what is the power output of the turbine in kW? ***[112.51kW]***

9. A reciprocating air compressor takes in $2\text{ m}^3/\text{min}$ at 0.11 MPa, 20 °C, which it delivers at 1.5 MPa, 111 °C to an after-cooler where the air is cooled at constant pressure to 25 °C. The power absorbed by the compressor is 4.15 kW. Determine the heat transfer in a) the compressor and b) the cooler. ***[-0.16 kW, 3.768 kW]***

CHAPTER 2 - SECOND LAW OF THERMODYNAMICS

Limitations of First Law:

- It does not ensure that the process will take place.
- It does not provide means for predicting the direction of processes

Second law

- *provide means for predicting the direction of processes,*
- *establishing conditions for equilibrium,*
- *determining the best theoretical performance of cycles, engines and other devices.*

Kelvin-Planck statement

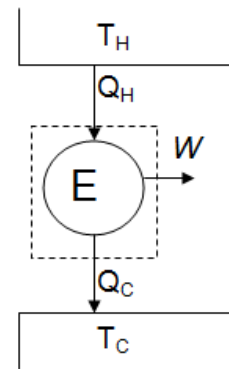
No heat engine can have a thermal efficiency 100 percent. ($Q_R \neq 0$).

Clausius Statement

Heat cannot flow from body at lower temperature to body at higher temperature spontaneously but it can be made possible with an external work.

Heat Engine:

- they receive heat from a high-temperature source
- they convert part of this heat to work
- they reject the remaining waste heat to a low temperature sink atmosphere
- they operate on a cycle



Thermal efficiency:

- ❖ Represent the magnitude of the energy wasted in order to complete the cycle.
- ❖ A measure of the performance that is called the thermal efficiency.
- ❖ Can be expressed in terms of the desired output and the required input

$$\eta_{th} = \frac{\text{Desired Result}}{\text{Required Input}}$$

- ❖ For a heat engine the desired result is the net work done and the input is the heat supplied to make the cycle operate.

The thermal efficiency is always less than 1 or less than 100 percent.

$$\eta_{th} = \frac{W_{net, out}}{Q_{in}}$$

where $W_{net, out} = W_{out} - W_{in}$

$$Q_{in} \neq Q_{net}$$

- ❖ Applying the first law to the cyclic heat engine

$$Q_{net,in} - W_{net,out} = 0$$

$$W_{net,out} = Q_{net,in}$$

$$W_{net,out} = Q_{in} - Q_{out}$$

- ❖ The cycle thermal efficiency may be written as

$$\begin{aligned}\eta_{th} &= \frac{W_{net, out}}{Q_{in}} \\ &= \frac{Q_{in} - Q_{out}}{Q_{in}} \\ &= 1 - \frac{Q_{out}}{Q_{in}}\end{aligned}$$

- ❖ A *thermodynamic temperature scale* related to the heat transfers between a reversible device and the high and low-temperature reservoirs by

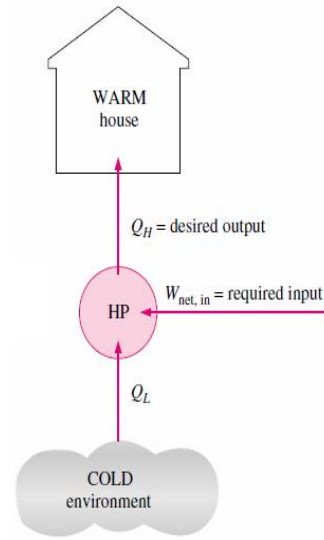
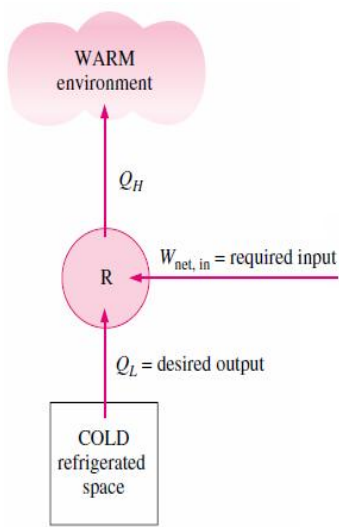
$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

- ❖ The heat engine that operates on the reversible Carnot cycle is called the Carnot Heat Engine in which its efficiency is

$$\eta_{th, rev} = 1 - \frac{T_L}{T_H}$$

Heat Pumps and Refrigerators

- A device that transfers heat from a low temperature medium to a high temperature one is the heat pump.
- Refrigerator operates exactly like heat pump except that the desired output is the amount of heat removed out of the system
- The index of performance of a heat pumps or refrigerators are expressed in terms of the coefficient of performance.



$$COP_R = \frac{Q_L}{W_{net, in}} = \frac{Q_L}{Q_H - Q_L}$$

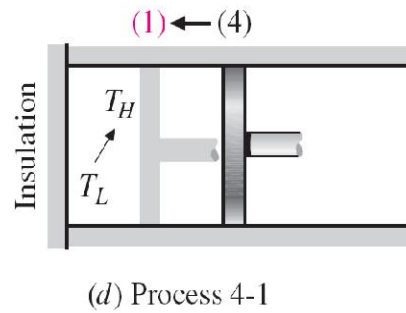
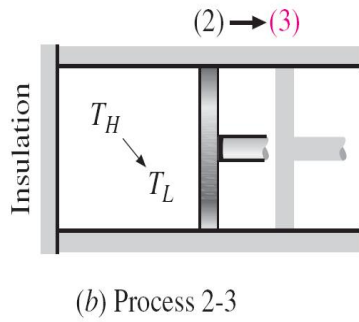
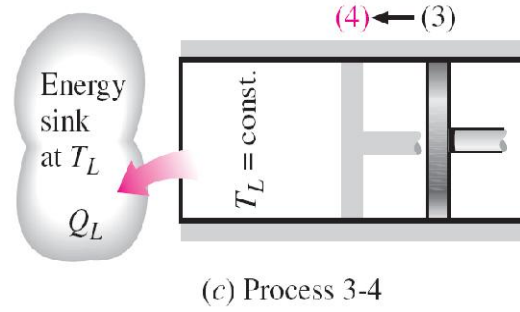
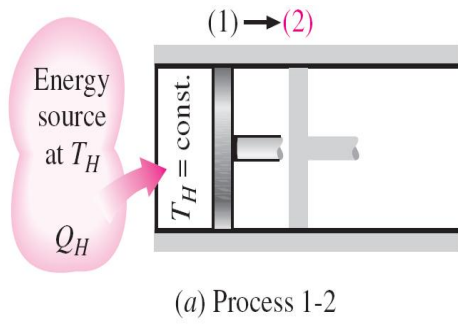
$$COP_{HP} = \frac{Q_H}{W_{net, in}} = \frac{Q_H}{Q_H - Q_L}$$

$$COP_{R, Carnot} = \frac{1}{T_H / T_L - 1} = \frac{T_L}{T_H - T_L}$$

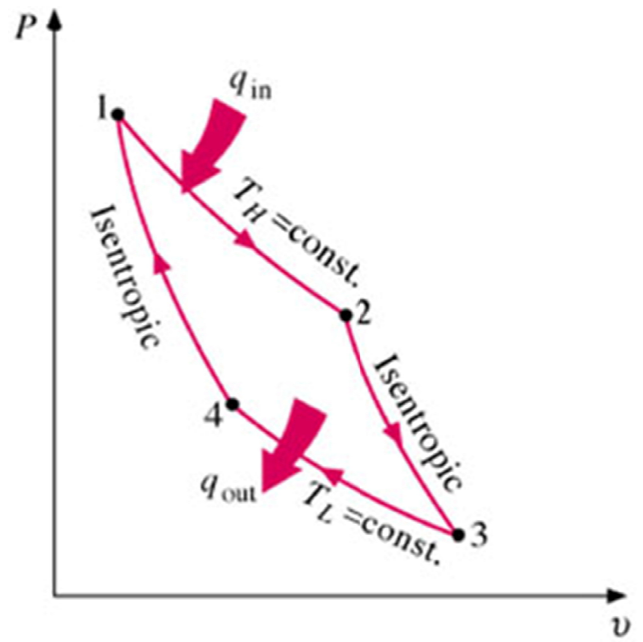
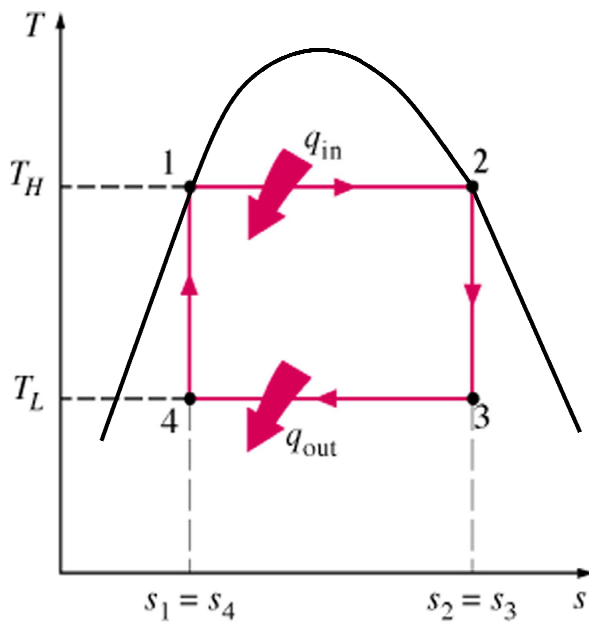
$$COP_{HP, Carnot} = \frac{1}{1 - T_L / T_H} = \frac{T_H}{T_H - T_L}$$

Carnot Cycle

Process	Description
1-2	Reversible isothermal heat addition at high temperature
2-3	Reversible adiabatic expansion from high temperature to low temperature
3-4	Reversible isothermal heat rejection at low temperature
4-1	Reversible adiabatic compression from low temperature to high temperature



Execution of Carnot cycle in a piston cylinder device



Carnot's Theorem: Among the many heat engines operating between the same temperature limits, only reversible heat engine (Carnot Engine) will have highest efficiency.

The thermal efficiencies comparison are as follows

$$\eta_{th} \begin{cases} < \eta_{th, rev} & \text{irreversible heat engine} \\ = \eta_{th, rev} & \text{reversible heat engine} \\ > \eta_{th, rev} & \text{impossible heat engine} \end{cases}$$

The coefficients of performance of actual and reversible refrigerators operating between the same temperature limits compare as follows

$$COP_R \begin{cases} < COP_{R, rev} & \text{irreversible refrigerator} \\ = COP_{R, rev} & \text{reversible refrigerator} \\ > COP_{R, rev} & \text{impossible refrigerator} \end{cases}$$

Problem #2.1: A power station contains a heat engine operating between two heat reservoirs, one consisting of steam at 100°C and the other consisting of water at 20°C . What is the maximum amount of electrical energy which can be produced for every Joule of heat extracted from the steam?

Solution:

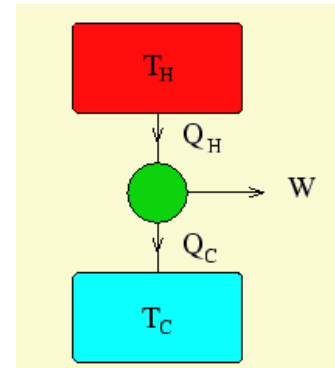
The maximum efficiency comes from using a Carnot (reversible) engine, for which $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$, and so

$$W = Q_H - Q_C = Q_H \left(1 - \frac{T_C}{T_H}\right)$$

With $T_H = 373\text{ K}$ and $T_C = 293\text{ K}$;

For every Joule of heat extracted from the hot reservoir ($Q_H = 1\text{ J}$)

$$W = 1 \times \left(1 - \frac{293}{373}\right) = 0.2144\text{ J}$$



Problem #2.2: When a fridge stands in a room at 20°C , the motor has to extract 500 W of heat from the cabinet, at 4°C , to compensate for less than perfect insulation. How much power must be supplied to the motor if its efficiency is 80% of the maximum achievable?

Solution:

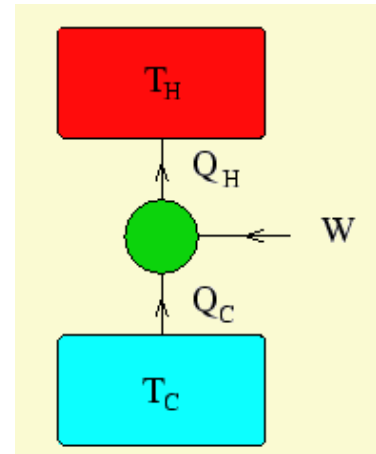
The maximum efficiency comes from using a Carnot (reversible) fridge, for which $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$. Here Q_C is known, and so

$$W = Q_H - Q_C = Q_C \left(\frac{T_H}{T_C} - 1 \right)$$

(In this question W , Q_C , and Q_H will refer to energy transfer per second, measured in Watts.) With $T_H = 293 \text{ K}$ and $T_C = 277 \text{ K}$, $Q_C = 500 \text{ W}$ gives $W_{\max} = 29 \text{ W}$.

However the real fridge works at 80% of the maximum efficiency,

$$\text{so } W = \frac{29}{0.8} = 36 \text{ W}$$



Problem #2.3: Two reversible engines A & B are arranged in series, E_A rejecting heat directly to engine, E_B . E_A receives 200 kJ at a temperature of 421°C from a hot source, while E_B is in communication with a cold sink at a temperature of 4.4°C . If the work output of E_A is twice that of E_B , find the efficiency of each engine.

Solution:

From Thermodynamic temperature scale we know that,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} ; \quad \frac{Q_2}{Q_3} = \frac{T_2}{T_3}$$

$$\text{But } \frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = \frac{T_1}{T_3}$$

$$\rightarrow Q_3 = 79.94 \text{ kJ}$$

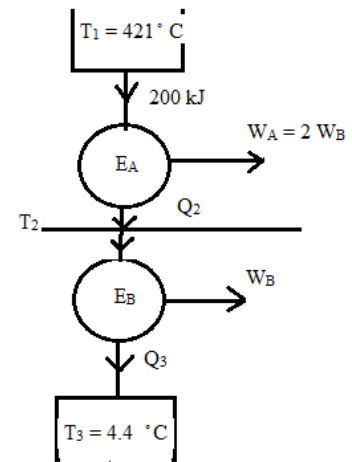
Since, $W_A = 2 W_B$

$$Q_1 - Q_2 = 2(Q_2 - Q_3) \quad \rightarrow Q_2 = 119.96 \text{ kJ}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \rightarrow T_2 = 143.26^\circ \text{C} ;$$

$$\eta_A = \frac{W_A}{Q_1} = 40\% \quad \text{where } W_A = Q_1 - Q_2$$

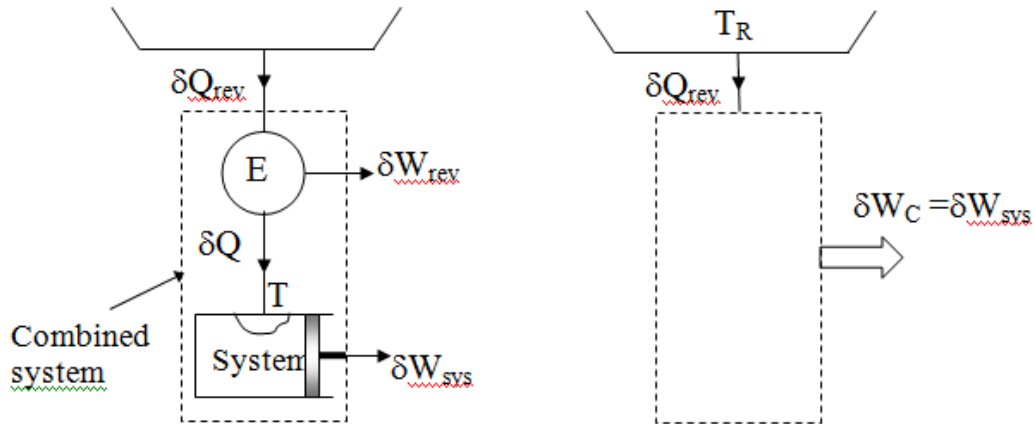
$$\eta_B = \frac{W_B}{Q_2} = 33.36\% \quad \text{where } W_B = Q_2 - Q_3$$



❖ **ENTROPY – A Measure of Disorder**

- Derivation of Entropy from Second Law

Consider the following combined system consisting of a Carnot heat engine and a piston cylinder system



First Law applied to the combined system

$$dE_C = \delta Q_{rev} - \delta W_C \quad ; \quad \delta W_C = \delta Q_{rev} - dE_C$$

For a Carnot Engine (reversible cycle)

$$\frac{\delta Q_{rev}}{\delta Q} = \frac{T_R}{T} \quad \rightarrow \quad \delta Q_{rev} = \delta Q \cdot \frac{T_R}{T}$$

Substituting $\delta W_C = \frac{\delta Q}{T} \cdot T_R - dE_C$

Let the system undergo a cycle while the Carnot cycle undergoes one or more cycles. Integrate over entire cycle. (Recall, system energy is a state property)

$$\oint \delta W_C = \oint \frac{\delta Q}{T} \cdot T_R - \cancel{\oint dE_C}$$

The net work for one cycle is

$$W_C = T_R \oint \left(\frac{\delta Q}{T} \right)_{boundary}$$

The combined system (cycle) draws heat from a single reservoir while involving work W_C

Based on K-P statement the combined system cannot produce net work output $\rightarrow W_C \leq 0$

$$\oint \left(\frac{\delta Q}{T} \right)_{boundary} \leq 0$$

This is the **Clausius Inequality** which is valid for all thermodynamic cycles, reversible or irreversible.

For a *reversible* cycle (no irreversibilities in the system) $\oint \left(\frac{\delta Q}{T} \right)_{int_{rev}} = 0$

Since the cycle integral of (dQ/T) is 0, the quantity dQ/T is a *state property*, it does not depend on the path (similar to $\oint dE = 0$).

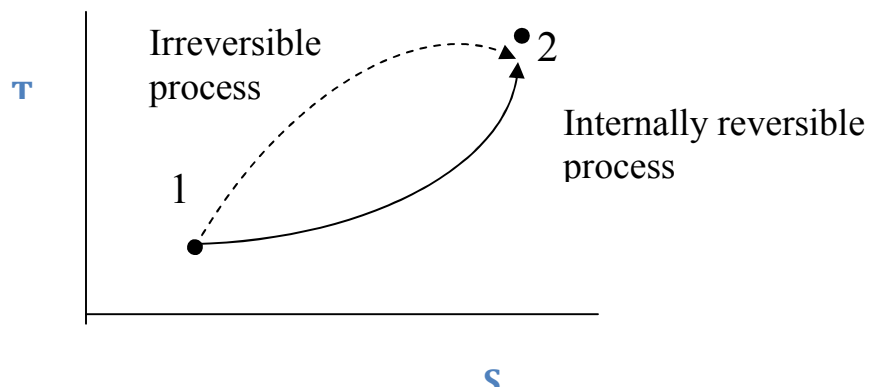
We call this as new property **entropy**, S , $dS = \left(\frac{\delta Q}{T} \right)_{int_{rev}}$ units : kJ/K

For a process where the system goes from state 1 to 2; $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{int_{rev}}$

Specific entropy $s = S/m$

$$s_2 - s_1 = \int_1^2 \left(\frac{\delta Q / m}{T} \right)_{int_{rev}} = \int_1^2 \left(\frac{\delta q}{T} \right)_{int_{rev}}$$

The entropy change between two specified states is the same whether the process is reversible or irreversible



To calculate $S_2 - S_1$ evaluate $\int \delta Q / T$ for the reversible path. Consider a cycle made up of an irreversible process followed by a reversible process

Applying Clausius Inequality to the cycle

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + \int_2^1 \left(\frac{\delta Q}{T} \right)_{int_{rev}} \leq 0$$

$$\int_1^2 \left(\frac{\delta Q}{T} \right)_b + (S_1 - S_2) \leq 0$$

$$S_2 - S_1 \geq \int_1^2 \left(\frac{\delta Q}{T} \right)_b$$

Remove inequality sign to get the entropy balance equation for a closed system

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_b + S_{gen}$$

Change in entropy
of the system

Entropy transfer to the
system by heat transfer

Entropy generated in the
system due to irreversibilities

$$S_{gen} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

For an **isolated system** (adiabatic closed system)

$$\Delta S_{isol} = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

Since $S_{gen} \geq 0 \rightarrow \square S_{isol} \geq 0$

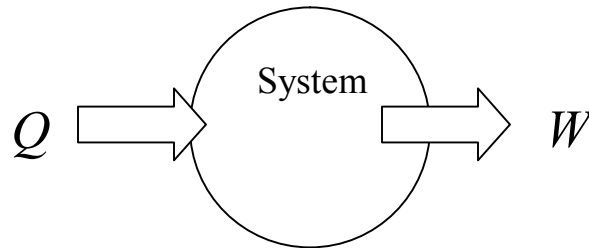
This is the **Increase in Entropy Principle** which simply stated says that for an isolated system the entropy always increases or remains the same

A process that is both adiabatic and reversible is referred to as **isentropic**, and for a closed

system, $\Delta S_{isentropic} = \int_1^2 \frac{\delta Q}{T} + S_{gen} = 0$.

Therefore, entropy is constant, $S_1 = S_2$.

Evaluation of Entropy Change in a Closed System



Apply First Law neglecting KE and PE effects, no shaft work

In differential form $du = \delta q - \delta w = \delta q - Pdv$

$$\delta q = du + Pdv$$

For an internally reversible process $ds = \frac{\delta q}{T}$ so

$$Tds = du + Pdv \quad \text{----- (1)}$$

recall $h = u + Pv \rightarrow du = d(h - Pv) = dh - Pdv - vdP$

substitute du into the above equation

$$Tds = (dh - Pdv - vdP) + Pdv \quad \text{----- (2)}$$

$$Tds = dh - vdP$$

These are the Gibb's equations (or simply Tds equations)

For a system undergoing a process from state 1 to state 2

$$s_2 - s_1 = \int_1^2 ds = \int_1^2 \frac{du}{T} + \int_1^2 \frac{P}{T} dv$$

$$s_2 - s_1 = \int_1^2 ds = \int_1^2 \frac{dh}{T} - \int_1^2 \frac{v}{T} dP$$

- **Entropy Change for an Ideal Gas**

For an ideal gas $du = c_v dT$, $dh = c_p dT$, $Pv = RT$

$$s_2 - s_1 = \int_1^2 \frac{du}{T} + \int_1^2 \frac{P}{T} dv = \int_1^2 \frac{c_v dT}{T} + R \int_1^2 \frac{dv}{v}$$

and

$$s_2 - s_1 = \int_1^2 \frac{dh}{T} - \int_1^2 \frac{v}{T} dP = \int_1^2 \frac{c_p dT}{T} - R \int_1^2 \frac{dP}{P}$$

If the specific heats c_p and c_v are taken as *constant*

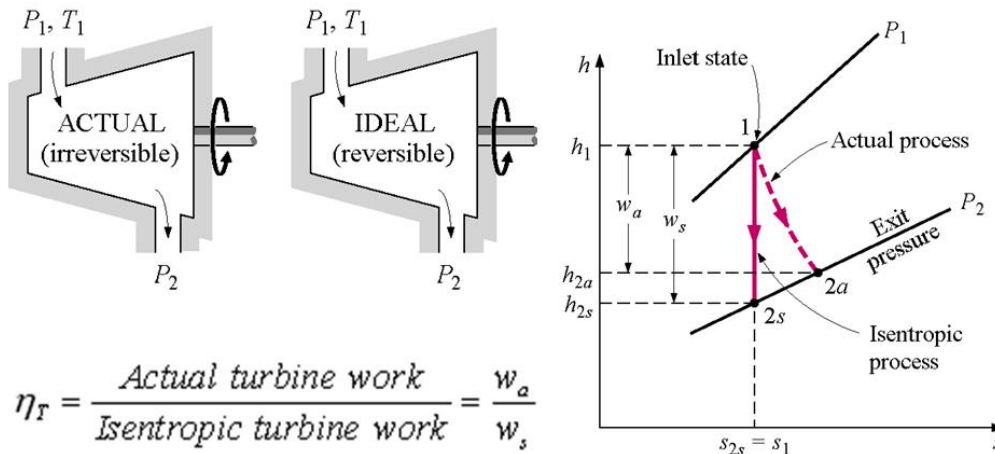
$$s_2 - s_1 = c_v \int_1^2 \frac{dT}{T} + R \ln \left(\frac{v_2}{v_1} \right)$$

$$s_2 - s_1 = c_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{v_2}{v_1} \right)$$

$$s_2 - s_1 = c_p \int_1^2 \frac{dT}{T} - R \ln \left(\frac{P_2}{P_1} \right)$$

$$s_2 - s_1 = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

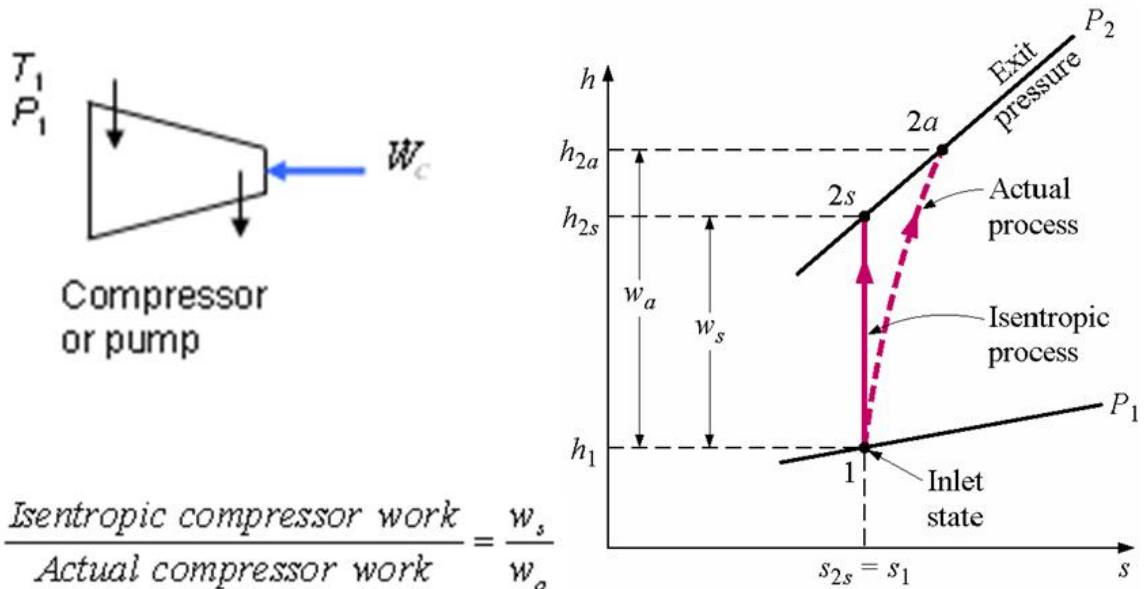
- **Isentropic Efficiency for Turbine**



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \equiv \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

- **Isentropic Efficiency for Compressor:**



$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_c \equiv \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

Problem#2.4: Air enters a compressor and is compressed adiabatically from 0.1 MPa, 27°C to a final state of 0.5 MPa. Find the work done on the compressor with an isentropic efficiency of 80 %.

Solution:

Applying SFEE for compressor,

$$W_c = \dot{m}(h_{2s} - h_1) \quad \rightarrow \quad W_c = \frac{\dot{W}_c}{\dot{m}} = (h_{2s} - h_1)$$

$$W_c = c_p(T_{2s} - T_1)$$

For isentropic process; $\left(\frac{T_{2s}}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_{2s} = 475.4 \text{ K}$

Then,

$$W_c = 1.005(475.4 - 300) = 176 \text{ kJ/kg}$$

$$(W_c)_{act} = \frac{W_c}{\eta_c} = 220 \text{ kJ/kg}$$

Problem #2.5: A piston–cylinder device contains 1.2 kg of nitrogen gas at 120 kPa and 27°C. The gas is now compressed slowly in a polytropic process during which $PV^{1.3} = \text{constant}$. The process ends when the volume is reduced by one-half. Determine the entropy change of nitrogen during this process.

Solution:

$$\text{Entropy Change, } s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

For a Polytropic process,

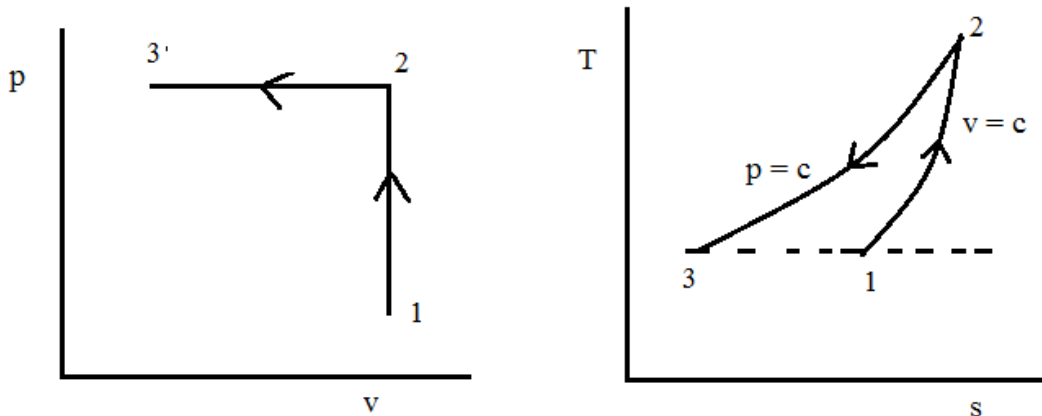
$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \text{So, } T_2 = T_1 * (2)^{0.3} = 369 \text{ K}$$

$$s_2 - s_1 = 0.718 * \ln\left(\frac{369}{300}\right) + 0.287 * \ln(0.5)$$

$$s_2 - s_1 = \mathbf{0.0596 \text{ kJ/kgK}}$$

Problem #2.6: Air at 20° C and 1.05 bar occupies 0.025 m³. The air is heated at constant volume until the pressure is 4.5 bar, and then cooled at constant pressure back to original temperature. Calculate a) the net heat flow from the air, b) the net entropy change.

Solution:



$$c_p = 1.005 \text{ kJ/kgK}; c_v = 0.718 \text{ kJ/kgK} \quad \text{----- Assumed}$$

$$\text{From gas equation, } m = \frac{p_1 v_1}{RT_1} = \frac{1.05 \times 10^5 \times 0.025}{287 \times 293} = \mathbf{0.0312 \text{ kg}}$$

For Process 1-2 (Constant Volume Process):

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \quad \implies T_2 = \mathbf{1255.7 \text{ K}} \quad (Q)_{v=c} = m c_v (T_2 - T_1) = \mathbf{21.56 \text{ kJ}}$$

For Process 2-3 (Constant Pressure Process):

$$(Q)_{p=c} = m c_p (T_3 - T_2) = -30.18 \text{ kJ} \quad \text{where } T_3 = T_1 \text{ and } -\text{'ve sign indicates heat loss}$$

$$\text{Entropy change in process 1-2, } s_2 - s_1 = m c_v \ln \frac{T_2}{T_1} = 0.0326 \text{ kJ/K}$$

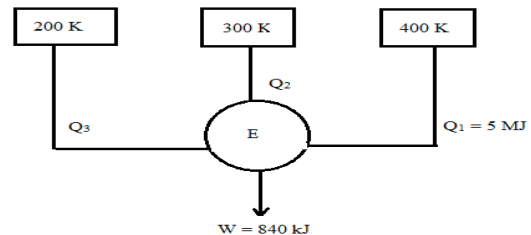
$$\text{Entropy change in process 2-3, } s_3 - s_2 = m c_p \ln \frac{T_3}{T_2} = -0.0456 \text{ kJ/K}$$

$$\text{Net entropy change, } (\Delta s) = s_1 - s_3 = -0.013 \text{ kJ/K}$$

Supplementary Problems for Practice

1. A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C . The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and -20°C . The heat transfer to the engine is 2 MJ and the net output of the combined engine and refrigerator plant is 360 kJ . Find the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C . Also find these values if the efficiency of the heat engine and COP of the refrigerator are each 40% of their maximum possible values.

2. A reversible engine as shown in Figure below during a cycle of operation draws 5 MJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.



3. The scales are so chosen that a reversible cycle plots clockwise as a circle on T - s plane. The minimum and maximum values of the temperature are 305 and 627 K and the entropy 1.23 and 2.85 kJ/k respectively. Find the cycle work and efficiency.
4. A gas initially at 603 K expands until its volume is 5.2 times the initial volume according to $p v^n = \text{constant}$. If the initial and final pressures are observed to be 8.5 bar and 1 bar , determine i) index of expansion, ii) workdone per kg of gas, iii) heat exchange per kg of gas, and iv) entropy change per kg of gas. Assume $c_v = 0.712 \text{ kJ/kgK}$ and $\gamma = 1.4$.
5. One kg of water at 273 K is brought into contact with a heat reservoir at 373 K . When the water has reached 373 K , find the entropy change of the water, the heat reservoir and the universe.